Underpricing, Volatility and Demand in U.S. Treasury Auctions*

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Abstract

Why are treasury auctions underpriced? This question has been the subject of much debate over nearly half a century. Much attention is given to how frictions increase underpricing. The two main streams of research consider either the role of financial frictions that result from the size of auctions or the role of auction design in the context of game theory. Both document that underpricing is increasing in volatility, but interpret volatility as amplifying the effect of frictions. In this paper, I separate out the effects of frictions and volatility on underpricing in U.S. Treasury auctions. I find that while both affect underpricing, the effect of volatility tends to be larger. This finding suggests that underpricing due to volatility is in fact compensation for risk, and that underpricing can persist in frictionless auction markets. This is a dramatic departure from convention because underpricing is viewed almost exclusively as an aberration from market efficiency.

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I. Introduction

Why are treasury auctions underpriced? This question has been the subject of much debate over nearly half a century. Much attention is given to how frictions increase underpricing. The two main streams of research consider either the role of financial frictions that result from the size of auctions (Lou, Yan, and Zhang, 2013) or the role of auction design in the context of game theory (Goldreich, 2007). Both document that underpricing is increasing in volatility, but interpret volatility as amplifying the effect of frictions. In this paper, I separate out the effects of frictions and volatility on underpricing in U.S. Treasury auctions. I find that while both affect underpricing, the effect of volatility tends to be larger. This finding suggests that underpricing due to volatility is in fact compensation for risk, and that underpricing can persist in frictionless auction markets. This is a dramatic departure from convention because underpricing is viewed almost exclusively as an aberration from market efficiency.

Underpricing can be measured as the return from buying at the auction price and reselling the same security on the day it is issued at the secondary market price. For example, 2-year notes earn an average cumulative return of 10.0 basis points (bps) and an average daily continuously compounded return of 3.11 (t=5.02) bps, in which daily return volatility is 7.02 bps. Return volatility is a measure for the riskiness of holding an asset. Investors require higher expected returns for holding riskier assets. Increases in expected returns due to increases in volatility are considered a risk premium (Engle, Lilien, and Robins, 1987). Every auction has an expected return from underpricing, which can result from frictions and volatility. To separate out these two effects, I model the return time series for a given maturity with a GARCH-M process. The basic mechanics of the GARCH-M involve two steps: (1) generate a forecast of return volatility for each auction, and (2) use the volatility forecast predict the expected return. The expected return consists of frictions and a risk premium, in which frictions are specified as the portion of expected returns that are not explained by volatility. I find that volatility explains the bulk of returns. For example, a 1 bp increase in volatility results in a 0.455 (t=3.59) bp increase in expected returns on 2-year note auctions. The remaining portion of returns explained by frictions is 0.15 (t=0.36), which is not different from zero. The implication is that volatility explains underpricing, and that underpricing can persist in the absence of frictions.

Imperfect capital mobility of end-investors and arbitrageurs is thought to prevent underpricing from being arbitraged away (Lou, Yan, and Zhang, 2013). Portfolio analysis theory and the capital-market model developed by Sharpe (1966) suggest that the optimal portfolio is one that seeks to maximize expected risk-adjusted returns (Sharpe ratio). The Sharpe ratio is the expected return per unit of risk. Sharpe’s theory would suggest that
end-investors and arbitrageurs decide to supply liquidity to auction markets based on the expected risk-adjusted return that results from underpricing. To test this conjecture, I use the estimated GARCH-M process to forecast an expected return and conditional volatility for each auction to create an expected risk-adjusted returns factor. I then regress auction demand (liquidity supply) on expected risk-adjusted returns controlling for other factors. I find that auction demand is positively and significantly related to expected risk-adjusted returns. For example, a 1 bp increase in expected risk-adjusted returns results in a $40.5 (t=3.20) billion increase in auction tenders of 2-year notes. The implication is that end-investors’ and arbitrageurs’ choice to supply liquidity depends on the returns that result from underpricing. Further, auction demand is limited by expected risk-adjusted returns. This link between underpricing and demand implies that underpricing may be a permanent part of debt issuance.

This framework provides an explanation for how lowering interest rates to the zero-lower-bound can help reduce underpricing and increase auction demand. When zero-interest-rate policies are binding, they can mechanically decrease return volatility. The reduction in volatility decreases the risk premium, which reduces underpricing. Contemporaneously, the reduction in volatility also increases expected risk-adjusted returns, which leads to increases in auction demand. For example, consider how 26-week bill auctions differed from 2003 to 2010: average weekly offerings increased from $16.2 to $27.9 billion per auction; daily returns decreased from 0.24 to 0.12 bps; risk-adjusted returns increased from 0.68 to 1.67; and the bid-to-cover ratio, which is the volume of bids (demand) divided by the volume of debt offered (supply), increased from 2.01 to 4.26.

Since 2009, the Treasury borrows roughly $8 trillion through auctions each year, in which underpricing of a single basis point represents a debt issuance cost of $800 million annually. Underpricing is thought to be increasing in auction size, as auction size is thought to increase the effects of frictions. Primary dealers take large pre-auction short positions to offset their risk of acquiring large auction allocations (Fleming and Rosenberg, 2007). These short positions can drive down prices before auctions. If end-investors and arbitrageurs supplied more liquidity, they would offset dealers’ short positions and eliminate underpricing. It is thought that imperfect capital mobility prevents them from supplying liquidity, and this effect is amplified by larger auctions (Lou, Yan, and Zhang, 2013). However, since 2009, we observe a paradox in the data, in which auction size increases but underpricing declines. The results in this paper help resolve this paradox. Liquidity supplied by arbitragers and end-investors is determined by expected risk-adjusted returns that result from underpricing. At the end of 2008, zero-interest-rate policies were binding on short-term Treasury securities. As a result, return volatility declined, which reduced underpricing and increased expected risk-
adjusted returns. The increase in expected risk-adjusted returns increased auction demand, resulting in large auctions with low underpricing and high demand.

**Related Literature.** These results are distinct from Lou, Yan, and Zhang (2013), who document that secondary market prices tend to decline in the 10-days before and increase in the 10-days after an auction. While they give compelling reasons for why prices decline before auctions, the reason why prices recover is less explicit. My results suggest that auction prices are systematically discounted relative to their issuance price, and the size of the discount depends on the expected volatility of the return. Price increases may appear to fan-out sometime over the 10-day post auction period because the settlement period varies from auction to auction. For example, 2-year notes settle in 3.72 days on average, but have settled in as few as 1 and as many as 9 trading-days later.

My results are distinct from Goldreich (2007), who considers underpricing as the difference between auction yields and contemporaneous when-issued yields, and explains underpricing as a result of the auction mechanism using game theory. Second, I show that the magnitude of underpricing relative to the secondary market post-issuance price is dramatically greater than for contemporaneous when-issued securities. Third, I show that underpricing and volatility are increasing in maturity, and therefore treat different maturities as separate time-series rather than as a cross-section of auctions. Fourth, he finds that underpricing and the bid-to-cover ratio are negatively correlated, suggesting that increases in competition reduce underpricing. I find similar results, but have an alternative explanation. I show that changes in expected risk-adjusted returns predict changes in the bid-to-cover ratio. My interpretation is that as expected-risk adjusted returns increase, auctions become more competitive. Fifth, while we both find a positive relationship between underpricing and volatility, his measure for volatility is the standard deviation of when-issued yields 30-minutes before an auction [closes]. My results use a forecast of return volatility, using past information only, which identifies the effect that volatility has on underpricing.

My findings are distinct from the findings of Nyborg, Rydqvist, and Sundaresan (2002). They consider discriminatory-price Swedish Treasury auctions from 1990-1994. A major difference is that Swedish Treasury auctions determine the supply after the auction closes. Similar to my results, they find that underpricing is increasing in volatility. However, they treat volatility as exogenous to underpricing. While they also find that underpricing is increasing in volatility, they pool their time-series and cross-section data, whereas my approach separates auctions by maturity. In an ad hoc way, they assume returns follow a random walk with constant drift, and model return volatility with an ARCH(2) that has the same parameters for all maturities. To account for the differences in volatility by maturity, they add a parameter to the ARCH equation that adds a fixed amount of volatility per unit
of duration. Finally, they also notice that auction demand is decreasing in volatility. My results explain why: increasing volatility can reduce expected risk-adjusted returns, which can decrease auction demand.

These results are also distinct from the on-the-run premium, which is the phenomenon that just issued on-the-run Treasury securities trade at a premium over just off-the-run securities (Amihud and Mendelson, 1986, 1991, Warga, 1992). This paper focuses on the increase in price from auction to issuance of the same security. Winning auction bidders commit to a price days in advance of settlement. The surprise is what the security is worth in the secondary market once it is issued. When the auction price is higher than the secondary market price, financial intermediaries will be forced to hold their inventory or sell at a loss. The risk is quantified by the volatility of the return from buying at the auction price and selling at the secondary market price because it represents the dispersion of the secondary market price relative to the auction price. When volatility is higher, the chance of taking a larger loss is also higher. The compensation for this risk is a discounted auction price, in which the discount is determined by the expected volatility of the return. Therefore, underpricing is compensation for risk.

II. Data, Definitions and Institutions

A. Data Sample

Data are obtained from the U.S. Treasury and Bloomberg. The sample period ranges from January 2000 to June 2016. The maturities included are 4-, 13-, 26-, and 52-week bills; 2-, 3-, 5-, 7-, and 10-year notes; and 30-year bonds. Auction result variables include the bid-to-cover ratio, total offering amount, total accepted, total tendered, and both auction and issuance dates. Pricing variables for bills include the auction high rate and the last traded discount rate in the secondary market on issue day. Pricing variables for notes and bonds include the price per $100, which includes accrued interest, and the last traded price in the secondary market on issue day. Bill yields are converted to prices by the formula given in appendix B of the auction regulations Uniform Offering Circular and Amendments (31 CFR Part 256).

The main analysis of this paper is completed on a subset of the data, in which the main determinant of inclusion is a constant frequency of auctions over the sample period. Weekly auctions of 4-, 13-, and 26-week bills are considered from July 30, 2001 through June 28, 2016, which aligns with the introduction of 4-week bills and allows for an apples-to-apples comparison. Monthly auctions of 2-year notes are considered from January 2000 through June 2016. The constant frequency of auctions allows for the application of many time series analysis techniques that would not be possible with irregular auction frequencies.\(^1\)

One observation of 26-week bills is dropped due to a missing observation in Bloomberg, which corresponds to CUSIP 912795QC8. The chosen subset of maturities is tested and found to exhibit autoregressive conditional heteroskedasticity (ARCH) effects, which allows for effective modeling of the time varying volatility and risk premium of each auction.

B. How Treasury Auctions Work

Since October 1998, all U.S. Treasury securities are issued through regular and predictably scheduled single-price sealed-bid auctions (Garbade, 2015). Months in advance of an auction, the Treasury makes a tentative announcement regarding future debt issuance. As the auction date approaches, the Treasury makes an official offering announcement specifying all pertinent information regarding the upcoming offering, such as the total offering amount, the auction closing time, and issuance (settlement) and expiration (maturity) dates. Bidding for the new security begins after the official offering announcement. Auction bids are submitted as tenders, which are price and quantity pairs.

Primary dealers do not need to hold funds in their account with the Federal Reserve

\(^1\)Additionally, one can extrapolate information from this subset and infer the main mechanism more generally to other maturities.
to submit tenders, nor do they need to until settlement. For this reason, holding a winning bid until settlement has no carry cost. Most tenders arrive just before the auction closes (Goldreich, 2007), and auction results are announced roughly two minutes later. The price is determined by the marginal bidder, as the last tender accepted determines the price paid by all winning bidders. If tenders at the stop out price exceed supply, bidders are awarded a reduced percentage of their quantity demanded. Day(s) later, the securities are issued and monies are collected from winning bidders to settle the transaction.

For example, figure I presents a hypothetical scenario in which the Treasury sells 80 units of debt through an auction closing on date $\tau$. Tenders arrive randomly until the auction closes, and are then sorted from highest to lowest price. Starting from the highest price, tenders are accepted until all 80 units of debt are sold. The last tender accepted determines $P^A_\tau$, which is the auction price paid by all winning bidders. In this case, $P^A_\tau$ is equal to $99.2$ and denotes the stop out price for the auction. Figure II depicts a general timeline of events from the official announcement to security issuance. After the auction price is determined, the securities are issued $d$ trading days later, and trade in the secondary market there after. $P^S_{\tau+d}$ denotes the last traded price in the secondary market on issue day.

<table>
<thead>
<tr>
<th>Tenders Submitted</th>
<th>Tenders Sorted</th>
<th>Tenders Accepted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>Quantity</td>
<td>Price</td>
</tr>
<tr>
<td>99.0</td>
<td>33</td>
<td>99.9</td>
</tr>
<tr>
<td>99.3</td>
<td>16</td>
<td>99.5</td>
</tr>
<tr>
<td>98.5</td>
<td>12</td>
<td>99.3</td>
</tr>
<tr>
<td>98.3</td>
<td>17</td>
<td>99.2</td>
</tr>
<tr>
<td>99.5</td>
<td>22</td>
<td>99.1</td>
</tr>
<tr>
<td>99.9</td>
<td>14</td>
<td>99.0</td>
</tr>
<tr>
<td>97.9</td>
<td>11</td>
<td>98.9</td>
</tr>
<tr>
<td>99.1</td>
<td>19</td>
<td>98.5</td>
</tr>
<tr>
<td>99.2</td>
<td>28</td>
<td>98.3</td>
</tr>
<tr>
<td>98.9</td>
<td>10</td>
<td>97.9</td>
</tr>
</tbody>
</table>

Figure I: Single-price sealed bid auction timeline. Suppose the Treasury sells 80 units of debt. Tenders are submitted in random order until the auction closes. Tenders are then sorted, and then accepted starting from highest to lowest price, until the total offering is sold. The last tender accepted determines $P^A_\tau = 99.2$, which is the price paid by all winning bidders.

Figure II: Announcement to issuance timeline. Tenders are submitted after the official offering announcement until the auction closes on date $\tau$. The price that all winning bidders will pay, $P^A_\tau$, is released roughly two minutes after the auction closes. The securities are issued and monies are collected $d$ trading days later. The secondary market price, $P^S_{\tau+d}$, is observed after the securities are issued.
C. Returns from Underpricing

Underpricing occurs when the auction price $P^A_\tau$ is lower than the secondary market price $P^{S}_{\tau+d}$, in which $\tau$ is the auction date of a security that is issued $d$ trading days later. One can think of underpricing as either a difference or as a return. Dealers’ typically purchase from the auction with the intent to resell in the secondary market (Bikhchandani and Huang, 1993). Therefore, it is appropriate to use returns to evaluate performance. We can think of a positive return as a measure of underpricing; the bigger the return, the greater the underpricing. Dealers’ view this return as remuneration for their services, but the Treasury views it as a debt issuance cost (Lou, Yan, and Zhang, 2013). Since there is no carry cost to holding a winning auction bid, the return is also an excess return. We can express this return as either the cumulative return $R_t$, or as the daily continuously compounded return $r_t$, in which $t$ indexes a sequence of auctions (weekly, monthly). Then,

$$R_t = \frac{P^{S}_{\tau+d} - P^A_\tau}{P^A_\tau}, \quad r_t = \frac{1}{d} \ln \frac{P^{S}_{\tau+d}}{P^A_\tau}.$$ (1)

<table>
<thead>
<tr>
<th>Security</th>
<th>$E[r_t]$</th>
<th>$t$-Stat</th>
<th>$\sigma_r$</th>
<th>Amn.%</th>
<th>$E[R_t]$</th>
<th>Obs.</th>
<th>Min</th>
<th>Mean</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-week bills</td>
<td>0.15</td>
<td>(5.91)</td>
<td>0.45</td>
<td>0.38</td>
<td>0.29</td>
<td>778</td>
<td>1</td>
<td>1.87</td>
<td>3</td>
</tr>
<tr>
<td>13-week bills</td>
<td>0.28</td>
<td>(6.06)</td>
<td>0.90</td>
<td>0.70</td>
<td>0.77</td>
<td>779</td>
<td>1</td>
<td>2.86</td>
<td>4</td>
</tr>
<tr>
<td>26-week bills</td>
<td>0.48</td>
<td>(6.28)</td>
<td>1.37</td>
<td>1.22</td>
<td>1.34</td>
<td>779</td>
<td>1</td>
<td>2.86</td>
<td>4</td>
</tr>
<tr>
<td>52-week bills</td>
<td>1.14</td>
<td>(4.34)</td>
<td>1.80</td>
<td>2.88</td>
<td>2.22</td>
<td>106</td>
<td>1</td>
<td>1.89</td>
<td>2</td>
</tr>
<tr>
<td>2-year notes</td>
<td>3.11</td>
<td>(5.02)</td>
<td>7.02</td>
<td>7.83</td>
<td>10.00</td>
<td>198</td>
<td>1</td>
<td>3.72</td>
<td>9</td>
</tr>
<tr>
<td>3-year notes</td>
<td>2.63</td>
<td>(3.35)</td>
<td>6.47</td>
<td>6.62</td>
<td>10.86</td>
<td>109</td>
<td>3</td>
<td>4.66</td>
<td>8</td>
</tr>
<tr>
<td>5-year notes</td>
<td>8.41</td>
<td>(4.61)</td>
<td>18.48</td>
<td>21.18</td>
<td>21.36</td>
<td>170</td>
<td>1</td>
<td>3.47</td>
<td>8</td>
</tr>
<tr>
<td>7-year notes</td>
<td>13.38</td>
<td>(4.25)</td>
<td>29.50</td>
<td>33.72</td>
<td>28.72</td>
<td>89</td>
<td>1</td>
<td>2.30</td>
<td>7</td>
</tr>
<tr>
<td>10-year notes</td>
<td>8.82</td>
<td>(3.35)</td>
<td>30.22</td>
<td>22.22</td>
<td>20.65</td>
<td>150</td>
<td>1</td>
<td>3.63</td>
<td>7</td>
</tr>
<tr>
<td>30-year bonds</td>
<td>22.01</td>
<td>(3.36)</td>
<td>78.98</td>
<td>55.47</td>
<td>40.49</td>
<td>102</td>
<td>1</td>
<td>2.73</td>
<td>6</td>
</tr>
</tbody>
</table>

Table I: Summary statistics by maturity from January 2000 to June 2016. The first column lists the type of security auctioned. Columns two through five report average daily continuously compounded returns with Newey-West (1987) adjusted $t$-statistics, return volatilities, and annualized returns assuming a 252 trading day calendar. Column six reports the average cumulative return. Column seven reports the number of observations in the sample. Columns eight through ten report the number of days to settlement.

Table I provides return summary statistics by maturity. Consistent with Nyborg, Rydqvist, and Sundaresan (2002), returns and volatility tend to be increasing in maturity. The exceptions are 3- and 7-year notes, but these securities were issued irregularly over the sample period. The comparing of maturities is interesting but tricky, as different maturities have.

\footnote{Henry Kaufman, a Salomon Brothers economist argued against single-price auctions and improving efficiency as it “provides no incentives to...dealers to help in the distribution process...” (Kaufman, 1973, 170).}
been auctioned at different times and with different frequencies over the sample period. For example, 52-week bills were not offered in the sample period until June 2008; after which volatility and underpricing declined. The longer duration securities have seemingly massive returns; 30-year bond auctions earn an annualized return of 55%. However, 30-year bond auctions are extremely risky, with a daily return volatility of 79 bps. Also, you can see that the number of days until settlement varies widely, which may explain why prices appear to increase over the 10-day period following an auction (Lou, Yan, and Zhang, 2013).

D. Debt Issuance and Asymmetric Demand at the Zero-Lower-Bound

Following the 2008 financial crisis, the Treasury more than doubled the annual volume of debt sold through auctions. Figure III plots the annual gross volume of debt issued. At the time, many pundits were concerned that the market would not be able to absorb the large increase in supply. There were also concerns about the effect that zero-interest-rate policies would have on demand. Would investors choose cash over a less liquid piece of paper (treasuries)? Concerns diminished as demand increased by more than the increase in supply. Figure IV plots the annual gross bid-to-cover ratio, which is the gross annual bids (demand) divided by the gross annual offerings (supply).

Surprisingly, the response in demand was asymmetric in maturities. Demand for securities with interest rates close to zero experienced a much more pronounced increase than securities further out in the term structure. Figures V and VI demonstrate this effect by plotting the bid-to-cover ratio for auctions of 13- and 26-week bills versus 10-year notes. Before interest rates were lowered to zero, the bid-to-cover ratio for short and long-term securities was roughly the same. You can see that after interest rates were lowered to zero, the
increase in the bid-to-cover ratio for short-term securities far surpassed that experienced by long-term securities. This is a puzzling effect, that lowering interest rates increases demand. Also, if this phenomenon were simply explained by investors reaching for yield, why would the increase in demand be asymmetric? My results provide an answer to this puzzle: lowering interest rates to the zero-lower-bound mechanically reduces the volatility of returns that result from underpricing, which thereby increase expected risk-adjusted returns and auction demand. Changes in demand in response to changes in expected risk-adjusted returns can be explained by the optimal portfolio theory described by Sharpe (1966).

III. A Model Relating Underpricing, Volatility and Auction Demand

It is well established that risk averse investors require compensation for holding risky assets. Buying Treasury securities from auctions carries the risk of bidding too much and suffering the winners curse. Additionally, auction participants commit to a price days before settlement, which increases the uncertainty of what the security will be worth once it is issued. The incentive for primary dealers to provide underwriting services to the Treasury is not due to the benevolence of bankers, but rather for the opportunity to make a profit. A profit occurs when primary dealers, or other financial intermediaries, can buy securities at a discount in the auction market, in order to resell them at a premium in the secondary market. The same mechanism increases auction demand from end-investors as well, as buying the security for less will always increase an investors return. What follows is a model relating underpricing, volatility and auction demand.

Consider a two asset world in which investors must hold their wealth in shares of either a risky or certain asset, in which the risky asset has normally distributed returns and the certain
asset has a certain return. The risky asset is analogous to purchasing Treasury securities from
the auction market, while the certain asset is analogous to auction nonparticipation. The
risk in holding the risky asset is measured by the volatility of returns, and the compensation
is a higher expected return, which is increasing in volatility (risk). Demand for the risky
asset depends on investors’ utility functions, which determine risk preferences. Suppose the
numeraire is the certain asset, which is perfectly elastically supplied with price equal to 1
and certain payoff \( \bar{R} \) (gross interest). Let the risky asset have price \( P^A \) with a random payoff
\( P^S \), in which the random payoff has mean \( \pi \) and variance \( \omega^2 \). Investors wealth \( W \) can be
expressed as an allocation between \( X \) shares of the risky asset and \( Y \) shares of the certain
asset such that

\[
W = P^A X + Y. \tag{2}
\]

The excess return \( R \) of the risky asset per dollar invested in risky shares is given by

\[
R = \frac{P^S}{P^A} - \bar{R}. \tag{3}
\]

Since there is no carry cost to holding a winning auction bid, we can let the certain payoff
equal 1, and rewrite the excess return as the arithmetic return

\[
R = \frac{P^S}{P^A} - 1. \tag{4}
\]

For a particular auction that closes on date \( \tau \) with holding period \( d \), the return takes the
same form as equation 1. Define the mean and variance of \( R \) such that

\[
E[R] \equiv \mu = \frac{\pi}{P^A} - 1, \quad Var[R] \equiv \sigma^2 = \frac{\omega^2}{P^A^2}. \tag{5}
\]

Suppose further that all investors have utility functions that exhibit constant absolute risk
aversion,\(^3\) and that investors only maximize end of period wealth so that only the first two
moments matter. Investors maximize expected utility by choosing an allocation of risky and
certain assets such that

\[
E[U] = \max_{X,Y} \ 2E[X P^S + Y] - bVar[X P^S + Y]. \tag{6}
\]

Utility is maximized by choosing

\[
XP^A = \frac{\mu}{b \sigma^2}. \tag{7}
\]

\(^3\)For example, exponential utility implies constant absolute risk aversion.
The optimal solution relates the number of shares demanded \( X \) and price \( P^A \) of the risky asset to the risk-adjusted return \( \frac{\mu}{\sigma^2} \), and risk aversion parameter \( b \). In financial markets it is common for the variance of returns to change over time, a property that is commonly referred to as volatility clustering. If we allow the variance of the risky payoff \( \omega^2 \) to vary from auction to auction, then the other variables will vary from auction to auction as well (Engle, Lilien, and Robins, 1987). Let \( t \) index a sequence of auctions. Relaxing the assumption of constant variance allows us to think of equation 7 as dynamic optimal solution,

\[
X_t P^A_t = \frac{\mu_t}{b\sigma^2_t}.
\]

\( (8) \)

### A. Impact of the Zero-Lower-Bound

Under normal economic conditions, price and demand typically adjust to changes in risk-adjusted returns to ensure that the asset is fully held in equilibrium (failed auctions are avoided). Since prices are inversely related to yields, lowering short term interest rates to the zero-lower-bound (ZLB) effectively places an upper bound on Treasury security prices. In fact, by rule of the Treasury, auction tenders for negative interest rates were not allowed at the ZLB. This rule implies that prices sufficiently close to the upper bound will exhibit inflexibility towards upward price movements. Let \( \bar{P}^A \) represent the upward inflexible price when the ZLB is binding. We can now rewrite the optimal solution as

\[
X \bar{P}^A = \frac{\mu}{b\sigma^2}.
\]

\( (9) \)

When prices are sufficiently close to the upper bound, what is the impact on the variance of \( R \)? Under normal economic conditions, investors will estimate a point forecast for the expected value of \( P^S \), which will typically have a variance of possible prices with a symmetric density about the point estimate. However, when prices are sufficiently close to the binding price, investors will estimate the expected value of \( P^S \), but the density of possible prices will be asymmetric, with more mass below the point estimate than above. This change in density reduces the range of possible prices, which lowers the variance of \( R \). The reduction in variance decreases the denominator of the risk-adjusted returns factor \( \frac{\mu}{\sigma^2} \), thereby increasing risk-adjusted returns. Since price is sufficiently close to the binding price, it cannot adjust upwards, so the increase in risk-adjusted returns leads to an increase in demand. In fact, this is exactly what we see empirically in the data. The ZLB reduced the variance of \( R \), increased risk-adjusted returns \( \frac{\mu}{\sigma^2} \), and increased auction demand \( X \).
IV. Model Formulation and Estimation Procedure

This section first develops a model for the mean and variance of returns that result from underpricing. Next, it outlines how to construct the expected risk-adjusted returns factor, which is then used to explain auction demand. It then concludes with an explanation of how parameters and structural breaks are estimated for the mean and variance equations.

A. Mean and Variance Equations

It is convenient to think of prices as following a stochastic difference equation. Let $r_t$ be the daily continuously compounded return from buying at the auction price and selling at the secondary market price, and let $t$ index a sequence of auctions (weekly, monthly) for a given maturity. Also, let $\mu_t$ be the conditional mean of $r_t$, let $\sigma_t^2$ be its conditional variance, and let $\sigma_t$ be its conditional volatility. The mean equation can be expressed as the combination of an expected return and a white noise process, such that

$$ r_t = \mu_t + \epsilon_t. \quad (10) $$

The expected return $\mu_t$, can be further decomposed into market frictions $\delta$, and a risk premium $\gamma \sigma_t$ (Engle, Lilien, and Robins, 1987), such that

$$ r_t = \delta + \gamma \sigma_t + \epsilon_t. \quad (11) $$

The risk premium is a function of the conditional volatility, in which increases in volatility lead to increases in the risk premium, and therefore increases in expected returns. This specification is directly related to underpricing, and implies that underpricing will be large when volatility is high. This reflects that investors’ demand a higher return when an asset is riskier. A positive and significant $\gamma$ indicates that underpricing is compensation for risk. If $r_t$ is scaled in basis points, then a 1 basis point increase in volatility leads to a $\gamma$ basis point increase in the risk premium. From the Treasury’s perspective, this directly quantifies the impact that volatility has on debt issuance costs. From an investors perspective, this is the amount of compensation required to hold the risky asset.

A nonzero $\delta$ could reflect either market frictions, inefficiencies, structural issues, or even the linearization of a nonlinear function. Generally, it represents the portion of returns that are not explained by volatility. There are several plausible reasons for a positive estimate, including the price impact of supply and demand shocks (Lou, Yan, and Zhang, 2013), short-squeezes of when-issued sellers (Jordan and Jordan, 1996), collusion (Goswami, Noe, and Rebello, 1996), or even barriers to entry. For example, a positive estimate could indicate a secondary market preference (Vayanos and Vila, 2009), in which investors prefer to purchase
Treasury securities from the secondary market to avoid the uncertainties associated with the auction process. This preference could make secondary market securities more expensive as demand shifts from the auction to the secondary market. One might expect that an efficient auction market has a $\delta$ of zero, as this would indicate all underpricing is compensation for risk.

In financial markets, the variance (volatility) of returns is often time-varying, in which large price changes beget large price changes, and small price changes beget small price changes (Mandelbrot, 1963). To capture these dynamics, let the variance of returns follow a GARCH($p, q$) process, which consists of a constant $\omega$, $p$-lagged conditional variances, and $q$-lagged squared innovations. For Treasury bills, volatility dramatically increases during the 2008 financial crisis, and it dramatically decreases when interest rates are at the zero-lower-bound. To capture these large changes, I allow for a volatility regime switch in the variance equation. This switch allows the constant to change, but keeps the GARCH parameters fixed. To avoid a model that overstates the role that volatility has on explaining returns, I also allow the constant in the mean to change with each volatility regime. Volatility regime $k$ is expressed as the indicator variable $S_k$. In addition, an ARMA($P, Q$) process is added to the mean equation to soak up any serial correlation: however, no significant ARMA terms are included in the best fit models. The full model to estimate can now be described as a volatility regime switching ARMA-GARCH-M (Engle, 1982, Bollerslev, 1986, Engle, Lilien, and Robins, 1987), such that

$$r_t = \sum_{k=1}^{K} \delta_k S_k + \gamma \sigma_t + \sum_{i=1}^{P} \phi_i r_{t-i} + \sum_{j=1}^{Q} \theta_j \epsilon_{t-j} + \epsilon_t,$$

$$\sigma_t^2 = \sum_{k=1}^{K} \omega_k S_k + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2,$$

$$\epsilon_t = \sigma_t \eta_t, \quad S_k = \begin{cases} 1, & \text{if } t \in S_k, \\ 0, & \text{otherwise.} \end{cases}$$

The innovation in the mean equation $\epsilon_t$ can now be defined as a function of the conditional volatility and a sequence ($\eta_t$) of mean zero independently and identically distributed random variables such that $E[\eta_t^2] = 1$. The conditional variance is determined by a constant, $\omega_k$, corresponding to volatility regime $k$, $q$-lagged squared innovations and $p$-lagged conditional variances such that $\omega_k > 0$ ($k = 1, 2, ..., K$), $\alpha_i \geq 0$ ($i = 1, ..., q$), $\beta_j \geq 0$ ($j = 1, ..., p$), and $\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1$. The general model consists of $K$ volatility regimes estimated by $K - 1$ structural breaks in the variance, so that the structural breaks change the level of
the variance, but do not affect the GARCH parameters. The estimated $\omega_k$ is multiplied by the indicator function $S_k$, which takes on the value of 1 when $t$ is in the volatility set $S_k$ and zero otherwise.

B. Auction Demand and Expected Risk-Adjusted Returns

Portfolio analysis theory and the capital-market model of Sharpe (1966) suggest that investors should seek a portfolio that maximizes expected risk-adjusted returns. Underpricing of auctions results in risk-adjusted returns to auction buyers. If auction demand varies with changes in expected risk-adjusted returns, then expected underpricing affects demand. To test the relationship between auction demand and expected risk-adjusted returns, we must first create an expected risk-adjusted returns factor. Step 1 is to estimate the GARCH-M model described above. Step 2 is to produce forecasts of the conditional mean and variance for each auction. Step 3 is to divide the expected return (conditional mean) by the conditional variance for each auction, in which the quotient is an expected risk-adjusted return. Step 4 is to regress an empirical measure for auction demand onto expected risk-adjusted returns controlling for other factors. If the relationship is positive and significant, conclude that auction demand depends on expected risk-adjusted returns. This implies that expected underpricing plays an important role in determining auction competitiveness and liquidity supply.

The choice to divide the expected return by the variance rather than the volatility (standard deviation) reflects the assumption that volatility helps to determine the conditional mean. For example, suppose that $\delta$ is equal to zero. Then the expected return will equal $\gamma \sigma_t$, and the expected risk-adjusted return $\frac{\mu_t}{\sigma_t}$ will equal $\frac{\gamma \sigma_t}{\sigma_t^2}$, which will equal $\frac{\gamma}{\sigma_t}$. This is similar to an expected Sharpe (1966) ratio, in which $\gamma$ is the expected return per unit of risk. As volatility increases, expected risk-adjusted returns will decrease, and theory (Sharpe, 1966) predicts that auction demand should decline as well. For a complete illustration, juxtapose this measure to one that divides the expected return by volatility. The expected risk-adjusted return will simply be the constant $\gamma$, which would imply that auction demand is also constant.

Consider now the regression of auction demand on expected risk-adjusted returns. Let $\chi_t$ represent the empirical measure of auction demand, $E_{t-1}\left[\frac{\mu_t}{\sigma_t^2}\right]$ be the expected risk-adjusted return, and let $Z_t$ be some vector of controls, such that

$$\chi_t = \beta_0 + \beta_1 E_{t-1}\left[\frac{\mu_t}{\sigma_t^2}\right] + Z_t \Gamma + \epsilon_t. \tag{12}$$

Notice that the expectation is formed with information from the previous auction, so the
information set is orthogonal to $\epsilon_t$. This implies that the effect that expected risk-adjusted returns have on auction demand is identified and the coefficient $\beta_1$ can be interpreted causally. The two empirical measures of auction demand considered are auction tenders (total number of bids) and the bid-to-cover ratio (auction tenders divided by auction size). For the regression of auction tenders, I control for auction size because primary dealers are required to bid on a pro-rata basis, and for serial correlation using a one-period lag of auction tenders. For the regression of the bid-to-cover ratio, the required bids of primary dealers will be captured by the constant, so I only control for serial correlation using a one period lag.

C. Parameter and Structural Break Estimation

Parameters for the GARCH-M are estimated by maximum likelihood. All estimations are performed using MATLAB’s constrained optimization, in which the negative log-likelihood function is minimized. The inverse of the Hessian matrix evaluated at the optimum is used as an estimate for the asymptotic covariance matrix, and standard errors are estimated as the square root of the diagonal elements of this matrix. Volatility regime switching models considered range from an ARMA$(0,0)$-GARCH$(0,1)$-M to an ARMA$(6,6)$-GARCH$(3,3)$-M. The innovations corresponding to the initial observations that are lost due to the number of lags in a given model are all set to zero. To avoid kinks and flat areas in the log-likelihood function, all models are estimated a minimum of 100 times with different random starting values for the parameters. In order for a solution to be considered valid, the chosen model and parameters must be found identically the same for at least one tenth of the estimations, and the Hessian matrix must be positive definite. The final model selected exhibits these characteristics and is found to have the lowest BIC (Schwarz, 1978). The parameters for the regression of auction demand on expected risk-adjusted returns are estimated by OLS, in which standard errors are adjusted according to Newey and West (1987) with a max lag determined by Andrews and Monahan (1992).

Structural breaks in the variance are estimated separately by maturity. First, the return $r_t$ is regressed on a constant to obtain a vector of estimated residuals $\hat{e}_t$, which are mean zero. The estimated residuals are then squared, and the squared residuals are then regressed on a constant using ordinary least squares. The methodology of Bai and Perron (1998, 2003) is then used to estimate global structural breaks. The number of structural breaks is determined by the modified Schwarz criterion LWZ (Liu, Wu, and Zidek, 1997). The full sample of observations is depicted as the volatility set $\{S\}$. Volatility regime $k$ contains the subset of observations $\{S_k\}$, which is determined by $K - 1$ structural breaks. For example, if 26-week bills have two structural breaks, then there are three mutually exclusive volatility sets $\{S_1, S_2, S_3\}$, which are complements that span the set $\{S\}$.
V. Empirical Results

In this section, I empirically examine the relationship between underpricing and volatility; and the relationship between auction demand and expected risk-adjusted returns. This section concludes with the estimated model parameters and volatility regime switch dates for each maturity. To reduce the impact of outliers, returns on Treasury bills are winzorized at the 1 and 99 percentiles.

A. Underpricing and Volatility

Recall that underpricing can be expressed as the return from buying at the auction price and reselling the same security on the day it is issued at the secondary market price, in which positive returns imply underpricing. Lou, Yan, and Zhang (2013) and Goldreich (2007) document that underpricing and volatility in U.S. Treasury auctions may be correlated. I add to this discussion by identifying the effect that volatility has on underpricing in uniform-price U.S. Treasury auctions, and estimate the percentage of underpricing that is explained by volatility. Table II presents these results.

![Table II: Volatility and Underpricing](image-url)

Table II: Volatility and Underpricing. The impact that volatility has on expected returns (underpricing) is estimated by $\gamma$, which determines the risk premium $\gamma \sigma_t$. A 1 basis point change in volatility leads to a $\gamma$ basis point change in expected returns. The second row reports the unconditional expected return in basis points. Row three reports $\hat{\mu}_1$, which is calculated by subtracting the risk premium forecast from each realized return and then taking the unconditional mean. By comparing the two means of 4-week bills, we can see that volatility explains 65% of underpricing. In the forthcoming model estimation section, we will see that most of the unexplained returns occur during the financial crisis.

First, the effect that volatility has on underpricing is captured by $\gamma$, which helps to determine the risk premium $\gamma \sigma_t$. For all maturities, $\gamma$ is positive and significant, indicating that underpricing is compensation for risk. Take 4-week bills for example, a 1 basis point increase in volatility leads to a 0.45 (t=4.36) basis point increase in expected returns. The next row reports the unconditional return $\hat{\mu}_1$ over the sample period. The third row reports $\hat{\mu}_2$, which is calculated by subtracting the risk premium forecast from each realized return and then taking the unconditional mean. By comparing the two means of 4-week bills, we can see that volatility explains 65% of underpricing. In the forthcoming model estimation section, we will see that most of the unexplained returns occur during the financial crisis.
Figure VII: Plots of returns $r_t$ (upper plots) and risk premium forecasts $\gamma \sigma_t$ (lower plots), in basis points by maturity. Black stemmed returns denote positive returns, red stems denote negative returns, and dashed lines denote estimated volatility regime switch dates. The risk premium is shaded by volatility regime $S_k$, and ZLB marks the actual zero-lower-bound date.

Figure VII plots realized returns $r_t$ and the estimated risk premium $\gamma \sigma_t$ for each auction by maturity. One can see that the risk premium moves with the volatility of returns, implying that volatility and underpricing move together. When volatility is high, underpricing is also high. Recall that $\sigma_t$ is the conditional volatility forecast of a return for a given auction, and is estimated using only past information. Both returns and the risk premium are scaled to basis points. There are obvious differences in volatility during the periods of economic expansion, financial crisis, and at the zero-lower-bound. Treasury bills have distinct volatility regimes ($S_1, S_2, S_3$), in which return plots are marked with a dashed line to indicate a regime switch date. Plots of the risk premium are shaded differently by volatility regime, and the dashed ZLB line marks the date of the actual zero-lower-bound. It may be important to note that these results do not rely on a regime switch, as 2-year notes have only one regime. Each maturity has its own pattern of underpricing and volatility, indicating that pooling maturities may result in biased results.
B. Auction Demand and Expected Risk-Adjusted Returns

In this section, I empirically examine the relationship between auction demand and expected risk-adjusted returns. Auction underpricing results in large excess returns to auction buyers, which are explained by the volatility of returns. Expected underpricing is therefore explained by expected volatility, which also determines expected risk-adjusted returns. Since an optimal portfolio is one that seeks to maximize expected risk-adjusted returns (Sharpe, 1966), auction demand is related to underpricing and volatility. This relationship gives the Treasury (any Treasury) both a way to understand, predict, and influence auction competitiveness, and a way to reduce underpricing. Underpricing can be reduced in ways that decrease the volatility of returns: one way to do this is by reducing the number of days between auction close and security issuance. This framework also provides a resolution to the supply shock paradox (Lou, Yan, and Zhang, 2013): that since 2009, auction size increased but underpricing declined. My results explain that lowering interest rates to the zero-lower-bound decreased the volatility of returns, increased expected risk-adjusted returns, and increased auction demand.

\[
\text{Tenders}_t = \beta_0 + \beta_1 E_{t-1}[\mu_t] + \beta_2 \text{Offer}_t + \beta_3 \text{Tenders}_{t-1} + \epsilon_t
\]

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Coeff.</th>
<th>t-stat</th>
<th>Coeff.</th>
<th>t-stat</th>
<th>Coeff.</th>
<th>t-stat</th>
<th>Coeff.</th>
<th>t-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-Week Bills</td>
<td>3.410***</td>
<td>(4.96)</td>
<td>1.090***</td>
<td>(3.64)</td>
<td>2.666***</td>
<td>(3.16)</td>
<td>40.50***</td>
<td>(3.20)</td>
</tr>
<tr>
<td>13-Week Bills</td>
<td>1.513***</td>
<td>(8.24)</td>
<td>1.280***</td>
<td>(4.51)</td>
<td>1.112***</td>
<td>(3.89)</td>
<td>1.173***</td>
<td>(7.15)</td>
</tr>
<tr>
<td>26-Week Bills</td>
<td>0.580***</td>
<td>(10.2)</td>
<td>0.760***</td>
<td>(15.4)</td>
<td>0.768***</td>
<td>(13.3)</td>
<td>0.623***</td>
<td>(12.4)</td>
</tr>
<tr>
<td>2-Year Notes</td>
<td>-11.8***</td>
<td>(-7.1)</td>
<td>-12.1***</td>
<td>(-4.0)</td>
<td>-9.41***</td>
<td>(-3.5)</td>
<td>-6.35***</td>
<td>(-2.9)</td>
</tr>
</tbody>
</table>

Table III: OLS regressions of auction tenders on expected risk-adjusted returns by maturity. For all maturities, the effect that expected risk-adjusted returns have auction tenders is positive and significant. For example, a 1 basis point increase in expected risk-adjusted returns leads to a $2.67 (t=3.16) billion increase in auction tenders of 26-week bills. The covariance matrix and resulting t-statistics are adjusted by Newey and West (1987), with a max lag determined by Andrews and Monahan (1992).

Table III presents results for the regression of auction tenders on expected risk-adjusted returns. For each maturity, expected risk-adjusted returns are positively and significantly related to auction tenders, suggesting that underpricing may in fact help the Treasury sell debt and avoid auction failures. We can interpret the estimated coefficient $\beta_1$ as the response in auction tenders to a 1 basis point change in expected risk-adjusted returns. For example, a 1 basis point increase in expected risk-adjusted returns leads to a $3.4 (t=4.96) billion increase in auction tenders of 4-week bills. This effect can be interpreted in light of the fact...
that the average auction size is 25 billion dollars over the sample period. Interestingly, the response of 2-year notes is much more sensitive to these changes, indicating that duration risk is important to auction bidders. The large adjusted $R^2$ suggests that a significant amount of variation in auction tenders is explained.

An alternative, and perhaps more popular measure for auction demand is the bid-to-cover ratio, which is the volume of auction tenders divided by the offering amount (auction size). Table IV presents results for the regression of the bid-to-cover ratio on expected risk-adjusted returns. Again, the results show a positive and significant relationship for each maturity, suggesting again that auction demand depends on expected risk-adjusted returns that result from underpricing. The estimated coefficient represents the response of the bid-to-cover ratio to a 1 basis point change in expected risk-adjusted returns. For example, a 1 basis point increase in expected risk-adjusted returns leads to an increase in the bid-to-cover ratio of 0.093 (t=7.10) for 4-week bills. Again, the response of 2-year notes is much more sensitive to changes in expected risk-adjusted returns.

\begin{table}
\centering
\begin{tabular}{lcccc}
\hline
 & 4-Week Bills & 13-Week Bills & 26-Week Bills & 2-Year Notes \\
\hline
$E_{t-1}[\sigma_t^2]$ & \textbf{0.093***} & \textbf{0.031***} & \textbf{0.085***} & \textbf{1.834***} \\
Bid-to-cover$_{t-1}$ & 0.762*** & 0.913*** & 0.894*** & 0.648*** \\
Constant & 0.478*** & 0.200*** & 0.231*** & 0.747*** \\
$R^2_{\text{adjusted}}$ & 0.755 & 0.913 & 0.923 & 0.740 \\
Observations & 777 & 777 & 776 & 197 \\
\hline
\end{tabular}
\caption{OLS regressions of the bid-to-cover ratio on expected risk-adjusted returns by maturity. For all maturities, the effect that expected risk-adjusted returns have on the bid-to-cover ratio is positive and significant. For example, a 1 basis point increase in expected risk-adjusted returns leads to a 0.085 (t=5.44) increase in the bid-to-cover ratio of 26-week bills. The covariance matrix and resulting t-statistics are adjusted by Newey and West (1987), with a max lag determined by Andrews and Monahan (1992).}
\end{table}

### C. Model Estimation by Maturity

Mean and variance equations, and volatility regime switch dates are separately estimated for each maturity. The results are reported in table V. What follows is a discussion of the empirical findings, with an emphasis on the similarities and differences between maturities.

Treasury bills are estimated to have three volatility regimes with two switch dates, but Treasury notes only have one regime and no switch dates. The first switch date occurs in 2007, around the onset of the 2007-2008 financial crisis, in which volatility greatly increased. Both 13- and 26-week bills experienced the switch around the August 6\textsuperscript{th} auction, while 4-week bills switched around the May 22\textsuperscript{nd} auction. The second switch date occurs in 2008,
around the time interest rates were approaching the zero-lower-bound, in which volatility greatly declined. Both 13- and 4-week bills experienced a switch around the November 3\textsuperscript{rd} and 4\textsuperscript{th} auctions, while 26-week bills switched around December 1\textsuperscript{st}. The estimated break dates can be considered in light of actual events.\textsuperscript{4} The second switch for all three maturities occurred after interest rates were reduced to 1.0 percent (October 29\textsuperscript{th}), and before interest rates were reduced to zero (December 16\textsuperscript{th}). This may suggest that interest rates at or below 1 percent may be sufficiently close the zero-lower-bound to have an effect on volatility, which has an effect on underpricing and auction demand. Inversely, raising interest rates above 1 percent may significantly increase volatility.

For a given volatility regime, the constant in the variance equation is monotonically increasing in maturity. This finding may suggest a term-structure of volatility, that perhaps could help to explain the term-structure of interest rates. For a given maturity, the constant in the variance equation corresponds to the period of moderate volatility during economic expansion, high volatility during the financial crisis, and low volatility when interest rates are near the zero-lower-bound (ZLB). The volatility during the financial crisis was drastically higher than either of the other two regimes. For example, the constant in the variance equation is 45/95/76 times greater than the constant at the ZLB for 4-/13-/26-week bills. The variance equations for 4-, 13-, 26-week bills and 2-year notes are described as a GARCH(0,1)-M, GARCH(0,2)-M, GARCH(2,2)-M, and GARCH(1,1)-M.

The constant in the mean equation for 2-year notes is not statistically different from zero, indicating that all of the returns that result from underpricing are explained by volatility. For Treasury bills, the mean equation has the same constant for both the periods of economic expansion and the ZLB, but a different constant during the financial crisis. The constants are monotonically increasing in maturity for a given regime. During the economic expansion and the ZLB, the constant is not different from zero for 4-week bills, and is close to zero (0.05 and 0.06 bps) for 13- and 26-week bills. During these times, returns are almost completely explained by volatility. However, during the financial crisis, the constant is quite large, reaching 1.16 (t=3.92) bps for 26-week bills. A nonzero constant implies that during the crisis, some frictions were at work. While this paper remains agnostic as to what caused this, there are several possible reasons discussed in section IV. Important to this paper, is that the constant during the crisis shows why volatility does not explain 100 percent of returns.

Since $\gamma$ and the risk premium were discussed at length earlier in this section, I do not discuss them further here. Also, no ARMA parameters were significant for any of the best fit models.

\textsuperscript{4}See, for example, https://www.stlouisfed.org/financial-crisis/full-timeline for a timeline of events around the financial crisis presented by the Federal Reserve Bank of St. Louis.
Mean Equation: \( r_t = \sum_{k=1}^{K} \delta_k S_k + \gamma \sigma_t + \sum_{i=1}^{P} \phi_i r_{t-i} + \sum_{j=1}^{Q} \theta_j \epsilon_{t-j} + \epsilon_t \)

\[
\begin{array}{cccc}
\delta_1 & 0.0085 & 0.0507 & 0.0620 & 0.1546 \\
 & (0.66) & (3.76) & (2.67) & (0.36) \\
\delta_2 & 0.3756 & 0.9290 & 1.1622 & \\
 & (3.61) & (4.08) & (3.92) & \\
\delta_3 & \delta_1 & \delta_1 & \delta_1 & \\
\gamma & 0.4457 & 0.2524 & 0.3636 & 0.4550 \\
 & (4.36) & (3.68) & (5.05) & (3.59) \\
\end{array}
\]

Variance Equation: \( \sigma_t^2 = \sum_{k=1}^{K} \omega_k S_k + \sum_{j=1}^{P} \beta_j \sigma_{t-j}^2 + \sum_{i=1}^{Q} \alpha_i \epsilon_{t-i}^2 \)

\[
\begin{array}{cccc}
\omega_1 & 0.0336 & 0.0568 & 0.1417 & 0.3626 \\
 & (10.1) & (6.59) & (4.56) & (1.90) \\
\omega_2 & 0.3308 & 1.1870 & 1.5295 & \\
 & (4.51) & (3.40) & (2.92) & \\
\omega_3 & 0.0073 & 0.0124 & 0.0201 & \\
 & (9.32) & (7.10) & (3.91) & \\
\alpha_1 & 0.4708 & 0.5877 & 0.1796 & 0.1994 \\
 & (5.10) & (6.66) & (3.31) & (3.97) \\
\alpha_2 & 0.4193 & 0.4193 & & \\
 & (5.52) & (5.52) & & \\
\beta_1 & 4.8e-6 & & & 0.8006 \\
 & (7.3e-5) & & & (20.1) \\
\beta_2 & 0.2919 & & & \\
 & (4.54) & & & \\
\end{array}
\]

Volatility Regimes:

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<tr>
<th>( S_1 )</th>
<th>First Auction</th>
<th>7/30/01</th>
<th>7/30/01</th>
<th>7/30/01</th>
<th>1/26/00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Last Auction</td>
<td>5/22/07</td>
<td>8/6/07</td>
<td>8/6/07</td>
<td>6/20/16</td>
<td></td>
</tr>
<tr>
<td>Observation(t)</td>
<td>1-303</td>
<td>1-315</td>
<td>1-314</td>
<td>1-198</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>( S_2 )</th>
<th>First Auction</th>
<th>5/30/07</th>
<th>8/13/07</th>
<th>8/13/07</th>
</tr>
</thead>
<tbody>
<tr>
<td>Last Auction</td>
<td>11/4/08</td>
<td>11/3/08</td>
<td>12/1/08</td>
<td></td>
</tr>
<tr>
<td>Observation(t)</td>
<td>304-379</td>
<td>316-380</td>
<td>315-383</td>
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<table>
<thead>
<tr>
<th>( S_3 )</th>
<th>First Auction</th>
<th>11/12/08</th>
<th>11/10/08</th>
<th>12/08/08</th>
</tr>
</thead>
<tbody>
<tr>
<td>Last Auction</td>
<td>6/28/16</td>
<td>6/27/16</td>
<td>6/27/16</td>
<td></td>
</tr>
<tr>
<td>Observation(t)</td>
<td>380-778</td>
<td>381-779</td>
<td>384-778</td>
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</tbody>
</table>

Table V: Parameter estimates for mean and variance equations by maturity with \( t \)-statistics in parenthesis. Results are scaled in basis points. Volatility regime switch dates are also reported.
VI. Conclusion

The evidence suggests that volatility explains underpricing and expected risk-adjusted returns explain auction demand. These findings provide a link between underpricing, volatility and auction demand. The interdependency between underpricing and auction demand should give regulators reason to pause when considering changes to the auction design. Eliminating underpricing that is due to frictions seems a tangible goal. However, reducing underpricing due to volatility can only be achieved by reducing volatility, which reduces risk. Because returns and variances scale by the number of trading days between auction and issuance, reducing the settlement period should result in a smaller risk premium, which will reduce underpricing and debt issuance costs.

Reducing underpricing to zero may not be possible. Consider what would happen if the Treasury required immediate settlement. The risk premium from auction to issuance would go to zero, but auction buyers would have to hold precautionary capital in case all of their bids are accepted. Holding precautionary capital would likely induce a carry cost, which would likely be offset by lower auction prices. The optimal auction design should try to minimize underpricing by minimizing risk, carry costs and frictions.
References


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