Credit Rating Changes, Information Acquisition and Stock Price Informativeness

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Abstract

How do rating changes affect investor behavior and price informativeness? In a model with endogenous information acquisition and investors with limited attention, we show that the returns to information and the informativeness of the price system increase after a downgrade, but should not experience a significant change after an upgrade. Using a sample of U.S. publicly traded firms that experienced a change in the rating between 2000 and 2013, we find empirical support for our predictions. Unlike previous studies, we show that rating changes do not only affect price levels but investors’ incentives to acquire and trade on information.

*Keywords: Credit Ratings, Stock Price Informativeness, Informed Trading, Noise Trading, Limited Attention*

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1 Introduction

In a market with costly information acquisition, the returns to information depend on the extent to which that information can be used without being completely revealed to uninformed investors. Yet, the release of otherwise private information by information intermediaries, such as credit rating agencies, may also affect the returns to private information. In particular, a change in a credit rating is typically linked to at least two important events. First, a release of private information about a firm’s risk almost exclusively available to sophisticated investors. Second, the response of investors that only trade based on the newly public information. Thus, a credit rating change, besides leading to the revelation of otherwise private information, may also lead to an increase in the trading of less sophisticated investors. Consequently, investors’ incentives to become informed and, in turn, the efficiency of the price system is affected by the release of such information. The purpose of this paper is to examine the manner in which a change in credit ratings affects investors’ incentives to acquire private information as well as the resulting informativeness of the price system.

To this end, we consider a model of noisy rational expectations, based on Grossman and Stiglitz (1980), in which we study the equilibrium around the (anticipated) announcement of a change in the credit rating of a firm. There are two assets, a risk-free one and a risky stock of the firm, whose true fundamental value is private but can be observed at a cost. There are three dates representing the time periods before and after the change in rating. The change in the rating is assumed to occur with probability one in the intermediate date.¹ Each investor rationally anticipates the trading equilibrium after the change in the rating and decides whether to incur the cost of becoming informed. Different from Grossman and Stiglitz (1980), we assume that those investors that remained uninformed are subject to limited attention concerns. Particularly, with some non-zero probability, an uninformed investor won’t be able to dedicate all the required attention to movements in the price of the risky asset, thus estimating an incorrect value for the asset and trading on it. A change in the credit rating is assumed to provide valuable information about the volatility of the assets of the firm as in Boot et al. (2006). In particular, consistent with the findings

¹The assumption that the rating change occurs with probability one in the second date simplifies the analysis but does not affect the result. We obtain similar results when the change in the rating is not perfectly anticipated. We discuss this in more details in Section 2.
of Kliger and Sarig (2000), we assume that a rating upgrade indicates a reduction in the risk of the firm while a rating downgrade indicates an increase the volatility of the stock. We let the probability of making wrong valuations due to limited attention depend on the uncertainty about the asset value, thus being linked with the credit rating of the firm.

Our analysis leads to two main results. First, relative to the case without a credit rating change, the proportion of investors that become informed increases following a credit rating downgrade. The intuition behind this result is as follows. A downgrade in the credit rating informs investors of greater uncertainty about the value of the asset, which by itself makes being privately informed more valuable. Besides that first effect, the greater uncertainty also means a greater probability of suffering from limited attention when trading. This, in turn, increases the amount of noise in the system, thus adding another benefit to becoming informed. These effects combined lead to an increase in the fraction of informed investors that surpasses the original increase in the uncertainty about the asset. Overall, this leads to an increase in the informativeness of the price system in equilibrium.

Our second result is that, relative to the case without credit rating change, the proportion of investors that become informed will not change after an upgrade in the credit rating of the firm. In this case, the precision of the available information about the risky asset increases, which makes it easier for all investors to learn about its value by looking to price movements. Thus, limited attention concerns tend to vanish. When this is the case, the decrease in the uncertainty about the asset associated to the upgrade in credit rating is an effect perfectly anticipated by rational investors when deciding to become privately informed. As a result of this, the fraction of informed investors decreases in the exact amount that cancels out the initial gain in precision provided by the rating. The net effect on the stock price informativeness is null.

There are two important considerations about our model worth to be highlighted. First, without the possibility of ending up with incorrect valuations due to limited attention from some of the uninformed investors, price informativeness would not vary in equilibrium after the change in the perceived volatility of the asset. In such a case, the effects over the incentives of becoming informed from the change in the uncertainty about the fundamentals would be rationally anticipated and canceled out in equilibrium. It is only when the possibility of some uninformed investors making a mistake in their estimations is allowed in the
model that we obtain a significant effect on the equilibrium stock price informativeness after a change in rating. This is because the decision to become informed has a feedback effect now, through the resulting amount of “noise traders” that emerge from that population of uninformed investors with wrong estimations. Those deciding to become informed make such decision not only based on the exogenous levels of noise in the market, but now taking into account that the resulting amount of uninformed agents proportionally determines the size of the overall noise trading that is associated to the change in rating. Secondly, though in practice credit ratings may contain some valuable information about the expected value of the fundamentals of the firm, we model them as a mere piece of information about the volatility of those assets, consistent with the view in Boot et al. (2006). Thus, a change in the rating has an impact on the noise present in the price system via the resulting change in the implied volatility of the assets (an informational effect nevertheless), while it also has an indirect impact through the effect on the probability of miscalculating the value of the firm given individual attention constraints.\textsuperscript{2}

To test the predictions of the model, we collect a sample of rating changes from 2000 to 2013. We compute four different measures of stock price informativeness. Our first measure was proposed by Roll (1988) and recently developed in Durnev et al. (2003) and Chen et al. (2007). This measure is computed on the basis of the correlation between a stock’s return and the return of the corresponding industry and market indexes. The intuition behind this measure is that if a firm’s stock return is strongly correlated with the market and industry returns, then the firm’s stock price is less likely to convey firm-specific information. Thus, the measure will be higher when the return on the stock is less correlated with the market and industry returns. Recent studies provide support for this as a measure of private information in stock prices (we discuss this in Section 4).\textsuperscript{3}

The second measure is the Easley et al. (2002) probability of informed trading (PIN). The PIN measure is based on a structural market microstructure model in which trades come from uninformed and informed traders. The PIN measure gauges the probability of informed trading in a stock, thus it is conceptually a sound measure for the amount of private information reflected in stock prices. Finally, we compute two additional measures.

\textsuperscript{2}Intrinsic in this assumption is the commonly held idea that the information about the assets of the firm that the CRA uses when setting its rating is privately known by at least some of the investors in the market.

\textsuperscript{3}Roll (1988) shows that this measure has very little correlation with public news, and thus it seems to capture private information.
of price informativeness which are related to the ownership structure of the firm. In particular, we compute the share of Active Mutual Fund Ownership and Long Term Institution Ownership. To the extent that such institutions are more likely to be sophisticated and more informed (see Harford et al. (2016) and Derrien et al. (2013)), a larger ownership held by such institutions is associated with a larger probability of an information-based trade and consequently a more informative price system.

We find evidence consistent with the predictions of the model. We find that firms that experience a downgrade exhibit a higher return Non Synchronicity and higher PIN following a credit rating downgrade. We also find that the proportions of Active Mutual Fund Ownership and Long Term Institution Ownership increase following a rating change. Thus, our results suggest that the frequency of informed trading and stock price informativeness increases following a credit rating downgrade.

Our empirical evidence is also consistent with the hypothesis that the proportion of informed investors trading a stock may or may not decrease following a credit rating upgrade. We find that the measures of stock price informativeness for the firms that experienced a rating upgrade are statistically indistinguishable to those of a set of observationally similar control firms. Thus, our results are consistent with the view that following a rating upgrade the effect on the informativeness of the stock price is weaker (if any), relative to the case of a downgrade.

The literature on the link between credit ratings and stock price informativeness is limited, the main exception being Odders-White and Ready (2006). Odders-White and Ready (2006) develop a theoretical model in which the uncertainty about future asset value is decomposed into some shocks that are simultaneously observed by all market participants, and some shocks that are initially observed by “informed” traders only. Their model predicts that firms with greater likelihood of facing privately observed shocks have lower credit ratings. They find evidence consistent with their model while using adverse selection measures to quantify market makers’ risk of facing an informed trader such as the Easley et al. (2002) PIN measure, or Hasbrouck (1991) information-based price impact, among others. That is, they provide evidence that the extent of adverse selection under which the stock is traded negatively affects the firm’s credit rating.
Our paper differs in two key aspects. First, while Odders-White and Ready (2006) focus on contemporaneous effects of adverse selection on the level of credit rating of a firm; in our paper we analyze the effect of changes to credit ratings on the resulting stock price informativeness. Second, they take the amount of adverse selection in the stock market as given to analyze its effect on the firm’s credit rating. In our model, the resulting amount of adverse selection is endogenous, with agents deciding to become privately informed while anticipating the effects of any given change in rating on how trading is undertaken. Thus, our predictions are about the effect of a change in rating on how stock price informativeness evolves around the announcement.

Another branch of the literature that is related to this paper, is the one that particularly looks at the effect of changes on credit ratings on stock prices. Goh and Ederington (1993), focusing on downgrades only, show that negative stock price reactions to downgrades are observed, though not for every downgrade announcement. Those that are anticipated, or those that are a consequence of a transfer of wealth from bondholders to stockholders, do not show a negative reaction. Hand and Leftwich (1992) look at the reaction of both, bond and stock prices, to the addition of the corresponding firm to Standard and Poor’s Credit Watch List, and to rating changes by Moody’s and Standard and Poor’s, and finds reliably non zero average excess returns for all these announcements. More recently, Kliger and Sarig (2000) test whether bond ratings contain pricing-relevant information by examining security price reactions to Moody’s refinement of its rating system, which allows for a cleaner empirical examination. They find that rating information does not affect firm value, but that debt value increases (decreases) and equity value falls (rises) when Moody’s announces better- (worse-) than-expected ratings. Naturally, this line of research supports the idea that a connection exists between changes in credit ratings and stock prices, but this literature has not examined the composition of investors that trade the stock as a reaction to credit rating changes, which is the focus of the present paper.

Our paper is also related to the vast literature that models the incentives for information acquisition in the market for a risky asset. The benchmark in this context is Grossman and Stiglitz (1980). In their model of noisy rational expectations, all investors are fully rational and perfectly anticipate the outcomes of the trading stage. This implies that, in equilibrium, price informativeness would remain invariant after an isolated change in the
prior uncertainty about firm value. In our model, the possibility of some behavioral reaction of some uninformed traders due to limited attention is shown to be fundamental in allowing the equilibrium price informativeness to vary with a change in rating, particularly leading us to predict the differential effect of an upgrade versus a downgrade. We think that this feature of our model is an important contribution to this literature.

The rest of the paper proceeds as follows. Section 2 describes the theoretical model. Section 3 provides the theoretical results that lead to our main empirical predictions. Section 4 describes the sample selection. Section 5 presents the empirical methodology that we use to test the main prediction of the model. Section 6 delivers our empirical results. Finally, Section 7 concludes.

2 The Model

The structure of our model is based on Grossman and Stiglitz (1980). The model consists of three dates, \( t = 0, 1, 2 \) (around the announcement of a change in rating), and three states of nature, \( \psi \in \{ H, M, L \} \). At \( t = 0 \) investors acquire information, at \( t = 1 \) they trade, and at \( t = 2 \) payoffs are realized.

2.1 Assets.

There are two assets: a risk-free asset, with return \( r \) normalized to 0, and a risky asset (a firm’s stock), whose fundamental value at \( t = 0 \) is a random variable

\[
\tilde{v} \sim N(0, \sigma_{\tilde{v},0}^2)
\]

The distribution of \( \tilde{v} \) is assumed to be common knowledge at \( t=0 \), and its realization \( v \) is publicly observed only after the trading stage of the model has ended (at \( t = 2 \)).

We let the volatility in the fundamental value of the firm be linked to the state of nature, \( \psi \). More specifically, we standardize things by assuming that at \( t = 0 \), the current state is linked to

\[4\text{We interpret the prior variance of the risky asset, } \sigma_{\tilde{v},0}^2, \text{ as the total amount of uncertainty about } \tilde{v} \text{ that is commonly perceived at } t = 0 \text{ (before any change in the rating is announced).} \]
$M$, which is associated to a variance $\sigma_{v,M}^2$. Similarly, if the state of nature were $H$ or $L$, the variance of $\tilde{v}$ would be $\sigma_{v,H}^2$ or $\sigma_{v,L}^2$, respectively. Importantly, we assume that

$$\sigma_{v,L}^2 > \sigma_{v,M}^2 > \sigma_{v,H}^2$$

2.2 Agents

In the model, there is a representative Credit Rating Agency (CRA), and infinitely many investors indexed over a set of measure one.

2.2.1 The CRA

The representative credit rating agency in our model has the objective of providing information about the risk of default of each firm that is analyzed.\(^5\) We model the CRA and the reported credit rating in a reduced form that results from a more complex equilibrium decision involving the maximization of profits of a CRA with market power (and potentially reputation concerns).\(^6\)

The CRA is assumed to learn about the state of nature $\psi$ at $t = 0$, and make an assessment about the probability of default for the firm based on that. We assume that it is on the CRA’s best interest to report the most accurate information as possible. Consequently, we let the announced rating, at $t = 1$, perfectly reveal the state of nature learned by the CRA ($\psi$). Formally, letting $\rho \in \{U, 0, D\}$ be the possible rating, we assume as common knowledge that

\(^5\)For our purposes, the key feature about credit ratings that we need in the model is the notion that they provide a public signal about the uncertainty in the value of the underlying asset. In practice, they (partially) provide information about the actual value of the asset, which does not play a key role in what we study (since we are not interested in the effect on price levels), and is unlikely to change the main results of the paper.

\(^6\)The assumptions that provide a microfounded justification for this reduced form can be found in Appendix C.
The characterization above immediately gives an important informational role to a (change in) credit rating, which we exploit in this model: Such a change is assumed to instantaneously affect the common beliefs about the value of the risky stock via its information about the variance of the associated probability distribution. Particularly, in the model, an upgrade ($U$) in credit rating leads to an instantaneous increase in the certainty about the possible value of the risky stock (or, equivalently, a decrease in the variance), from $\sigma^2_{v,M}$ to $\sigma^2_{v,H}$, while a downgrade implies a decrease in the precision of what is initially believed about the risky stock, from $\sigma^2_{v,M}$ to $\sigma^2_{v,L}$. This is consistent, for example, with Kliger and Sarig (2000), who find a negative relation between the direction of the announced credit rating changes and the implied volatilities (from option prices) for the corresponding stocks.\textsuperscript{7} This assumption is also consistent with what the Credit Rating Agencies claim to consider in their methodology to estimate credit ratings.\textsuperscript{8}

We also assume that at $t = 0$ it is public knowledge that a change in rating will be formally announced at $t = 1$. It is also assumed to be common knowledge the direction and magnitude of the change, which particularly implies that we consider the case of either an upgrade or downgrade separately. Though in practice there is no full certainty about the announcement of change in rating in the future (or even about the exact date of the announcement), information leakages, in general, result in the change in the credit rating being widely anticipated. This is a particularly valid observation for the days around the

\textsuperscript{7}Furthermore, Back and Crotty (2015), in a model a la Kyle (1985a), show that when considering the markets for bonds and stocks of a firm, the sign of the correlation of returns on debt and equity is the same as the sign of the cross-market Kyle’s lambda. The sign is positive or negative depending on whether investors’ private information is about the mean (level) or volatility (risk) of the firm’s assets, respectively. They empirically show that cross-market lambdas are significantly large and positive, implying that informed investors are generally acquiring private information about the expected value of the assets more than their volatility. This is consistent with our intuition that a change in rating should be much more valuable to rationally informed investors as a source of information about the riskiness of the assets ($\sigma^2_v$) than as information about the mean value of those assets.

\textsuperscript{8}For evidence on this, the reader can check the stated methodology available at each of the well known CRAs’ websites.
announcement of a change in rating. This assumption simplifies the analysis but does not affect our results. In particular, we could allow for a small chance that investors end up being wrong about this change and the results should still hold, just weaker than with full certainty about the change in rating.

2.2.2 Investors.

Preferences. All investors in the economy (informed and uninformed) are initially assumed identical, with risk neutral preferences and maximizing expected utility from terminal wealth. When trading in the stock market, investors face transaction costs that we assume quadratic in the amount being traded.

Endowments and individual budget. Each agent in the economy is endowed at time $t = 0$ with randomly assigned amounts of both risk-free asset and risky stock of the firm, which are respectively denoted (for a generic agent $i$) $B_{i0}$ and $X_{i0}$. The price of the risk free asset is 1, while the (endogenous) risky stock price is denoted by $P$. With this, each agent’s available wealth at the time that trading occurs would be $\tilde{W}_{i0} = B_{i0} + X_{i0}P$. The net supply of the risky stock is normalized to 0.

Limited attention. Investors are assumed to be subject to limited attention. Particularly, this limitation applies to the learning that investors can do about the value of the risky asset by studying price movements. We link the severity of this limitation to the uncertainty about the value of the risky asset (and thus, to the credit rating of the firm). Specifically, we do this in a reduced form, via two assumptions. First, at the time that trading occurs each investor will need to use effort (or, equivalently time, or some limited cognitive capacity) to learn from price movements. With probability $\lambda$ an exogenous event that deviates too much attention from this learning task (e.g. some personal matter, other

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10Similarly, cases in which a change in rating occurs in the opposite direction to what is commonly believed are extremely rare events, supporting the idea of public agreement about the direction of the change. Furthermore, though surprises in the magnitude of the changes in credit ratings are less rare, they are not commonly observed either and, given that credit ratings are coarse, those differences between what is expected and what is actually announced, when they occur, tend to be small.

11In considering limited attention, we follow Kahneman (1973), who specifies that it is a necessary consequence of the vast amount of information available in the environment, and the limits to information processing power of individuals. Thus, attention must be selective and requires effort, implying a substitution of cognitive resources from other tasks. There are multiple applications of this friction in economics and finance like, for example, Hirshleifer and Teoh (2003).
trading opportunities, etc.) may occur at that time. We assume that the investor is not able to perfectly determine whether this actually happened to him at the time of trading. Investors understand though, that if they were subject to such exogenous informational event their resulting estimated valuation for the risky stock would be incorrect. To simplify things, we consider that each (uninformed) investor that is subject to such an event ends up having the same, purely uninformative valuation for the asset $z \sim N(0, \sigma_z^2)$, independently distributed from any other random variable in the model.\(^{12,13}\)

Second, we acknowledge that more uncertainty about the asset implied that more of the investors's attention capacity will be needed to correctly learn from prices, making it more likely to face an event that deviates too much of his attention.\(^{14}\) At the same time, we allow for cases in which the uncertainty about the asset is low enough that the probability of making a mistake associated with limited attention becomes zero. More specifically, we assume the existence of a threshold level of variance $\bar{\sigma}^2$, such that limited attention is not a concern for any investor when the variance of beliefs about $\tilde{v}$ is below the threshold. Formally, this is modeled by assuming that the probability $\lambda$ that an investor gets an incorrect valuation $z$ is a function of the perceived variance about the asset value characterized by

$$
\lambda(\sigma_v^2) = \begin{cases} 
g(\sigma_v^2) \in (0, 1], & \text{for all } \sigma_v^2 > \bar{\sigma}^2, \
0, & \text{otherwise}, \end{cases}
$$

where $\sigma_v^2 \in \{\sigma_{v,M}^2, \sigma_{v,L}^2, \sigma_{v,L}^2 \}$, and $g(\cdot)$ being an increasing function.

In what follows, we standardize the analysis by considering the case where $\sigma_{v,M}^2 = \bar{\sigma}^2$. Thus, under this case, in the original state $M$ limited attention does not have an effect on

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\(^{12}\)Assuming the same noisy valuation $z$ for all these investors allows us to avoid dealing with correlation terms that are not relevant to the question that we are studying in this paper. Also, assuming that this valuation is at least partially informative about the value of the asset seems more realistic, but that would significantly increase the complexity in the model. For our results to hold, we need any non-zero level of error in these valuations, and so taking the extreme case of completely noisy valuations is a justified simplification.

\(^{13}\)With this, we are assuming that, on average, this valuation is normalized to zero. Particularly, since this is associated to the time of a change in rating, which can be an upgrade or downgrade, the reader may think that valuations should adopt a specific sign on average, related to the direction of the change in rating. It is not absolutely clear that this is the case in practice and, more importantly, adding this feature to the model would not change our results, which are based on the extra (noisy) trading volume created by these investors, independent of the direction in which they are trading. Adding a sign to the expected value in $z$ would end up only affecting the price level of equilibrium.

\(^{14}\)In assuming this, we have a more microfounded model of limited attention in mind, in which, at each point in time, investors allocate their limited attention capacity to different tasks. Intuitively, in such a model, we are thinking that properly learning about a very uncertain random variable requires more of this attention than learning about a variable with low variability.
the behavior of any investor. Importantly, since $\sigma_{v,M}^2 > \sigma_{v,H}^2$, limited attention will not be a binding concern either for investors if the CRA announces an upgrade in the credit rating of the firm. It is only for the case of a downgrade that the variance can go up enough to make the probability of getting an incorrect valuation strictly positive.

This last normalization is not a mere simplification of the model. It actually helps us capture in the clearest way as possible what we think is the key difference between more or less uncertainty about the asset value associated with downgrades versus upgrades.

**Information acquisition.** At $t = 0$, before any other event occurs (but understanding that a change in rating will be formally announced at $t = 1$), each agent has the option to invest an amount $k$ to become privately informed. A privately informed investor, more broadly, invests in a more sophisticated information processing technology. This, on one hand, allows them to learn about the actual realization of $\tilde{v}$. Besides that particular piece of private information, we assume that the more sophisticated processing technology that they acquire also allows them to be less exposed to limited attention concerns. More formally in the model, these investors have a higher threshold variance, $\overline{\sigma}_I^2 > \overline{\sigma}^2$ determining when the probability of making a mistake in analyzing information is non-zero. To simplify things, throughout the model we assume that informed investors are sophisticated enough to not actually face the constraint of limited attention. In terms of the model, this means that $\overline{\sigma}_I^2 > \sigma_{v,L}^2$ always hold. Those investors who decide to incur the cost $k$ will be called informed investors, while those that rationally decide not to acquire this more sophisticated information technology are called uninformed investors.

To summarize the role of limited attention in the model, informed investors do not face the concerns associated with limited attention, but uninformed investors may face it depending on the variance of $\tilde{v}$. Furthermore, for those uninformed investors, the probability of making a mistake due to limited attention satisfies $\lambda(\sigma_{v,L}^2) \geq \lambda(\sigma_{v,M}^2) \geq \lambda(\sigma_{v,H}^2)$, where equalities can only occur when at least two of these probabilities are equal to zero.

**Type of investors and individual valuations of the risky asset.** Informed investors’ demand for the risky asset, as a result of their individual maximization of expected utility, depends on the privately learned fundamental value $v$ and the observed stock price $P$. 


For any investor who, at \( t = 0 \) decided to remain uninformed, there are two possible scenarios: with probability \( (1 - \lambda(\sigma_{v,\psi}^2)) \), the investor will make a correct (i.e. rational) estimation of the value of the firm by learning from the observed stock price, and with probability \( \lambda(\sigma_{v,\psi}^2) \) the investor will make a mistake.

To ease the notation, in what follows we call “rational uninformed investors” (RU) those uninformed investors who behave rationally and learn from price, whereas those who end up getting a wrong estimation of firm value are called “incorrect uninformed investors” (IU). Rational uninformed investors behave as it is standard in these type of models, conjecturing the other investors’ trading strategies and learning from the observed price. Hence, rational uninformed investors’ demands will depend on the observed price in equilibrium through the learning about the fundamental value of the firm, which in turn determines their individual valuations for the risky stock (and hence whether and how much they want to buy or sell of it).

Incorrect uninformed investors are assumed to not learn from the price, but form their (purely noisy) individual valuation from an incorrect model estimation.\(^{15}\) Thus, their individual demands for the risky stock will only depend on the price through the comparison between their exogenous valuation and the observed price.

Lastly, as in Grossman and Stiglitz (1980), we also assume the standard exogenously given aggregate noise trading in the market for the risky stock. In the model, we denote it by a random variable \( \tilde{\varepsilon} \sim N(0, \sigma_{\varepsilon}^2) \), independent of any other random variable in the model.

### 2.3 Timeline

At each date \( t = 0, 1, 2 \) we assume a number of events occurring sequentially as described below.

At \( t = 0 \), the state of nature is \( M \), and all investors start the day with a common prior \( \tilde{v} \sim N(0, \sigma_{\tilde{v},M}^2) \). The CRA learns about the new state of nature (either \( H \) or \( L \)). The announcement of the new rating (to occur at \( t = 1 \)) is perfectly anticipated by investors at this time. Based on the expectations of a change in the rating and the corresponding

\(^{15}\)We could assume that they learn relatively less than those called “rational”, and the result would be analogous. What we intend to model is that some of the uninformed investors overly pay attention to some other event thus, at least partially, making wrong estimations about the value of the firm’s risky stock.
trading environment, each investor privately decides whether to incur the cost \( k \) of becoming informed (before the change in rating is announced). As a result of that, a fraction \( \alpha \) of the population of investors will be informed, and \((1 - \alpha)\) uninformed. The uninformed investors know that there is a probability \( \lambda \) (which can be zero, depending on the rating) that they will make a wrong estimation about the value of the firm’s stock due to their limited attention, but decide to take that risk to avoid the cost \( k \) of privately acquiring information.

At \( t = 1 \), the change in rating is announced and trading occurs. Informed investors trade based on their private information and the observed equilibrium stock price, rational uninformed investors trade based on their rationally updated beliefs once they see the change in rating and the resulting stock price. Incorrect uninformed investors trade based on the uninformed valuation they form from a mistaken model estimation. Aggregate noise trading is the fourth component of total trading activity at that time. Equilibrium stock price emerges from market clearing.

Finally, at \( t = 2 \), after trade has occurred, the realization of \( \tilde{v} \) is publicly observed and so is each agent’s final wealth.

### 2.4 Definition of Equilibrium

An overall equilibrium in this game requires both, the market for information and the market for the risky asset to be in equilibrium while being consistent with each other. Formally, we define an overall equilibrium sequentially. First, we define an equilibrium at the trading stage:

**Definition 1 (Equilibrium in Trading Activity)** For any possible case of a change in rating, an equilibrium is defined by a linear price function and demand functions of informed and uninformed agents of the economy such that:

1. The price function supports the market clearing for the risky stock.
2. Rational uninformed investors conjecture a linear equilibrium price function that is correct.
3. All investors in the market submit a demand for the risky asset that maximizes their individual expected utility given the equilibrium price.

With that, we can proceed to define and overall equilibrium for the whole game. Thus, for any possible announced rating, we have that:

**Definition 2 (Overall Equilibrium)** An overall equilibrium is defined by

1. A market for the risky asset being in equilibrium at \( t = 1 \) (Equilibrium in Trading Activity) and,

2. A fraction \( \alpha^* \in [0, 1] \) of investors that choose to become informed at \( t = 0 \) as a result of their individual maximization of expected utility at that time, taking into account the equilibrium behavior of all investors on the path of equilibrium in trading activity at \( t = 1 \).

### 3 Results and Empirical Predictions

Before describing the main results of our theoretical model for a given change in rating, we introduce a benchmark scenario against which the equilibrium outcome when a change in rating occurs is compared.

#### 3.1 Benchmark equilibrium

In order to have a clearer interpretation of the differential effect of a change in rating, we consider the case in which there is no change in rating as a benchmark scenario. It is easy to see that in such a case, our model is analogous to the traditional results of equilibrium in Grossman and Stiglitz (1980), with no limited attention. Since the derivation of this equilibrium is standard (and analogous to the equilibrium with a change in rating), we directly state the expression determining the resulting fraction of privately informed investors in this benchmark scenario, \( \alpha^*_M \), when the state of nature remains being \( M \):

\[
\alpha^*_M = \frac{\sqrt{\sigma^2 \sigma^2_{\pi,M} - 2kc}}{2k} \tag{1}
\]
Following Kyle (1985a), we measure the informativeness of the stock price in equilibrium by computing $Var[\tilde{v}|P]$. Thus, for the particular case of this benchmark, price informativeness in equilibrium would be

$$Var[\tilde{v}|P] = \sigma_{v,M}^2 \left( \frac{c^2\sigma_{\xi}^2}{(\alpha_M^*)^2\sigma_{v,M}^2 + c^2\sigma_{\xi}^2} \right).$$

### 3.2 Overall equilibrium with a change in rating

#### 3.2.1 Equilibrium in Trading Activity

The model is solved in two steps. First, the equilibrium trading activity is derived given a fraction of informed investors, $\alpha$. Second, we study the existence of an overall equilibrium (endogenizing the information acquisition stage). It should be noticed that the existence and characterization of any potential equilibrium in trading strategies depends on the direction of the credit rating change. Nonetheless, in what follows we derive the equilibrium in trading activity in a generalized way, that is, for any value of announce rating $\rho$.

First, we proceed to describe the general maximization problem of any agent of the economy (informed, rational uninformed, or incorrect uniformed) given a fraction of informed investors, $\alpha$, an expected fraction $(1-\alpha)\lambda$ of investors being uninformed investors that trade based on incorrect valuations, while the remaining investors also being uninformed though rationally learning from the equilibrium price.\(^{16}\) As can be noted, the form of the maximization problem applies to any type of investor in the economy (given the assumed identical preferences), just differing in the valuation for the risky stock of each of them, which in turn depends on the information that each type of agent considers.

Given $\alpha$ (and $\lambda$) and the considered change in rating, any agent $i$ will choose a demand $D_{i1} \equiv (X_{i1} - X_{i0})$ to maximize its final wealth, which is defined as

$$\tilde{W}_{i1} = \tilde{W}_{i0} + X_{i1}(\tilde{v} - P) - c\frac{D_{i1}^2}{2},$$

where $c$ corresponds to a parameter measuring the intensity with which total quadratic

\(^{16}\)Unless otherwise needed, we refer to a generic $\lambda$ to simplify notation.
transaction costs increase in the amount of stock being traded. The last expression describes that the agent starts with some initial wealth \( \tilde{W}_{i0} \), which can then be augmented or diminished depending on the realized value of the risky asset (and how much of that risky stock the agent ends up demanding), and to which the transaction costs are finally subtracted to compute the agent’s final wealth. Thus, for rational uninformed investors, the expected utility from the final wealth (conditional on their available information: \( P \) and \( X_{i0} \)) is represented by:

\[
E[\tilde{W}_{i1} - \frac{c}{2}D_{i1}^2 | P, X_{i0}] = W_{0i} + X_{i1}(E[\tilde{v}|P, X_{i0}] - P) - \frac{c}{2}(X_{i1} - X_{i0})^2
\] (2)

This is the objective function (to be maximized) of any rational uniform investor \( i \), when choosing an optimal traded amount \( D_{i1}^{RU} = (X_{i1}^{RU} - X_{i0}^{RU}) \) or, equivalently, optimal demand \( X_{i1}^{RU} \). The first-order condition is obtained by differentiating (2) with respect to \( X_{i1}^{RU} \). This leads to:

\[
X_{i1}^{RU} = X_{i0}^{RU} + \frac{(E[\tilde{v}|P, X_{i0}] - P)}{c}
\] (3)

In a similar way, for any incorrect uninformed investor \( i \) the valuation of the fundamental is assumed to be exogenously given by the realization \( z \). The rest of the maximization problem of this type of investor is the same as the one just described above for the rational uninformed one. With this, the optimal demand for an incorrect uninformed investor is given by

\[
X_{i1}^{IU} = X_{i0}^{IU} + \frac{(z - P)}{c}
\] (4)

Finally, and analogous to the previous types, we can derive the optimal demand of risky stock for an informed investor \( i \), noticing that we have assumed that informed investors observe the actual realization of \( v \). Therefore, after applying first order condition to his maximization of expected final wealth, the optimal demand for an informed investor is:

\[
X_{i1}^{I} = X_{i0}^{I} + \frac{(v - P)}{c}.
\] (5)

Having derived the generalized form of the optimal demands for each possible investor
in the market, two considerations are worth mentioning before proceeding to derive an equilibrium in trading activity. First, for the different possible changes in the rating of the firm, rational uninformed investors will correctly consider the sources of noise contained in the observed price function in order to learn about the value of the fundamental via a Bayesian updating of beliefs. Second, though all individuals in the model are assumed to be price takers (thus not being able to strategically manipulate the price with their individual demands), informed investors rationally consider the trading behavior of both types of uninformed investors when deciding their optimal trading strategies.

In order to find an equilibrium in the trading activity, we need to first consider the market clearing condition relevant to the problem:

\[ \alpha X_1^I + (1 - \alpha)[\lambda X_1^{IU} + (1 - \lambda)X_1^{RU}] + \varepsilon = 0, \]

where we assume that the net supply of the risky asset is zero. Replacing the form of the optimal demands for informed and uninformed investors previously derived into this market clearing condition, the market clearing condition can be expressed as:

\[ \alpha \frac{(v - P)}{c} + (1 - \alpha) \left[ \lambda \frac{(z - P)}{c} + (1 - \lambda) \left( E[\tilde{v}|P, X_{io}] - P \right) \right] + \varepsilon = 0, \] (6)

In order to find an equilibrium, we conjecture a linear price function of the form:

\[ P(v, \alpha, \lambda) = \beta v + \gamma \varepsilon + \eta z, \] (7)

where \( \beta, \gamma, \) and \( \eta \) are coefficients to be found in order to sustain the existence of an equilibrium. Particularly, this is the price function that rational uninformed investors conjecture when learning about the value of the fundamental from the observed price. In equilibrium, their conjecture (i.e. the estimated coefficients in the linear pricing rule) will be correct.

In deriving the results on the equilibrium in trading activity at \( t = 1 \), we assume that a feasible fraction \( \alpha \) of informed investors in the population resulted from the interaction in the market for information at \( t = 0 \). We defer to the next subsection the conditions that
ensure the feasibility of the resulting $\alpha$. The following result shows that a unique equilibrium in the trading stage (with a linear pricing rule) exists for a given fraction $\alpha$ of informed investors (and other given primitives of the model), and provides its characterization.

**Lemma 1 (Existence and Uniqueness of Equilibrium in trading activity.)** For any given $\alpha$ fraction of informed investors in the population, there exists an equilibrium where the linear price function is described by coefficients

$$
\beta = \frac{\alpha^2 \sigma^2_{v,\psi}(1 - \lambda(1 - \alpha)) + \alpha(c^2 \sigma^2_{e} + (1 - \alpha)^2 \lambda^2 \sigma^2_{z})}{\alpha^2 \sigma^2_{v,\psi} + c^2 \sigma^2_{e} + (1 - \alpha)^2 \lambda^2 \sigma^2_{z}}
$$

$$
\gamma = \frac{c}{\alpha} \beta, \text{ and } \eta = \frac{(1 - \alpha)}{\alpha} \lambda \beta,
$$

where $\psi \in \{H, L\}$ represents the new state of nature as a result of the change in the credit rating (upgrade or downgrade), and with optimal demands defined by $X_{1I} = X_{0I} + \frac{(v - P)}{c}$, for any given informed investor, $X_{1RU} = X_{0RU} + \left(\frac{\beta \sigma^2_{v,\psi} \beta \sigma^2_{v,\psi} + \gamma^2 \sigma^2_{e} + \eta^2 \sigma^2_{z} - 1}{\beta \sigma^2_{v,\psi} + \gamma^2 \sigma^2_{e} + \eta^2 \sigma^2_{z}}\right) \frac{P}{c}$, for any rational uninformed investor, and $X_{1IU} = X_{0IU} + \frac{(z - P)}{c}$, for incorrect uninformed investors. Furthermore, the equilibrium is unique over the class of equilibria with linear price functions.

Proof: See Appendix.

The lemma above provides a tractable functional form for the price and the demands in equilibrium for both cases, when the change in the rating is an upgrade or a downgrade. We now endogenize the information acquisition decision of rational investors, the level of informativeness potentially available in this market as well as the actual information that can be extracted by uninformed investors from the price in equilibrium. To further ease the exposition, we simplify the notation and avoid the superscript $\psi$, since the main differences associated to its value is accounted later on, when studying the overall equilibrium for both cases.

### 3.2.2 Overall Equilibrium (Endogenous information acquisition)

Understanding how the equilibrium arises in the trading stage at $t = 1$, investors can compute expected utilities at $t = 0$ under both cases: if they become informed and trade on that information, or if they remain uninformed, and with probability $\lambda$ naively trade
on incorrect valuations, and with probability \((1 - \lambda)\) rationally learn from the equilibrium price.

The way in which an equilibrium in the information acquisition stage is found follows a specific sequence. First, we assume that such an equilibrium exists (i.e. that there exists an \(\alpha^*\) of equilibrium for which a marginal investor would be indifferent between becoming informed or not). Naturally, for different values of the fraction of informed agents the benefits for an individual investor of becoming informed change, in the same way as it is discussed in Grossman and Stiglitz (1980). Second, we characterize how such \(\alpha^*\) of equilibrium would look like, if it existed. After that, we derive conditions under which its existence and uniqueness are guaranteed.

The following result characterizes the fraction of informed investors in equilibrium, \(\alpha^*\), if it exists:

**Lemma 2 (fraction of informed investors in equilibrium.)** Whenever it exists, an equilibrium in the information acquisition stage of the problem is characterized by

\[
\alpha^* = \frac{\lambda^2 \sigma_z^2}{(1 - \lambda) \sigma_v^2} - \sqrt{\left((\sigma_z^2 + \lambda \sigma_z^2)N(\lambda, k, c, \sigma_v, \psi) - \lambda^2 \sigma_z^2 (\sigma_{v, \psi}^2 - c^2 \sigma_{v, \psi}^2 + \lambda^2 \sigma_z^2)\right)}{\left(\sigma_{v, \psi}^2 + \lambda^2 \sigma_z^2 - N(\lambda, k, c, \sigma_v, \psi)\right)},
\]

(8)

where \(N(\lambda, k, c, \sigma_v, \psi) \equiv \frac{(1 - \lambda) \sigma_{v, \psi}^4}{(\sigma_{v, \psi}^2 + \lambda \sigma_z^2 - 2kc)}\).

**Proof:** See Appendix.

The above result shows that having a fraction \(\lambda\) of the uninformed investors suffering the consequences of limited attention plays an important role in the provision of incentives to become informed. The resulting fraction of informed investors in equilibrium takes into account that those incorrect traders effectively become an added component of noise trading after the change in rating, which is good news for those deciding to become informed. Besides that direct effect, ex-ante knowing that there is a positive chance of behaving naively after creates a cost of not becoming informed. The magnitude of such cost depends on how much informed investors may profit from the acquired information in equilibrium, which in turn depends on how much noise is eventually observed in the market. Thus, the magnitude of the mentioned cost of not becoming informed endogenously results from the interaction
of the λ component with the other sources of noise in the market \((\sigma_{\psi}^2, \text{ and } \sigma_{\varepsilon}^2)\). These benefits of becoming informed are only relevant when \(\lambda \in (0, 1)\), and are added to the rest of standard costs and benefits that are considered in Grossman and Stiglitz (1980). In the end, \(\alpha^*\) corresponds to the marginal investor that is indifferent between becoming informed or not. Later on, after proving the existence and uniqueness of the equilibrium in our model, we provide some characterization on how \(\alpha^*\) depends on \(\lambda\).

We now determine the combinations of parameters (if any) that allow for an equilibrium to exist. In order to do that, first define \(\omega(k) = 2kc\), and with that, we proceed to define three specific values that are used to prove and characterize the existence of the equilibrium at this stage. Specifically, we define:

- \(k \equiv k\) such that \(\omega(k) = \lambda\sigma_z^2 + \theta\sigma_{\psi,\psi}^2\), where \(\theta = \frac{\lambda(\sigma_{\psi,\psi}^2 + c^2\sigma_\varepsilon^2 + \lambda^2\sigma_z^2) + \lambda\sigma_{\psi,\psi}^2}{\lambda^2\sigma_z^2 + c^2\sigma_\varepsilon^2 + c^2\sigma_{\psi,\psi}^2} \leq 1\),
- \(\bar{k} \equiv k\) such that \(\omega(\bar{k}) = \lambda\sigma_z^2 + \sigma_{\psi,\psi}^2 > \omega(k)\), and
- \(k^*\) as the value of \(k\) for which \(\alpha^* = 1\), whenever it exists.

An equilibrium in the information acquisition stage is only obtained for a subspace of the combination of parameters. The values of \(k\) defined above play an important role since they allow us to characterize the subspace of parameter combinations for which an equilibrium exists based on the value of \(k\) only. That is what next result specifies.

**Proposition 1 (Existence and Uniqueness of an overall equilibrium.)** There exists a unique \(k^*\) for which \(\alpha^* = 1\), determined by \(k^* = \frac{\sigma_{\psi,\psi}^2 + \lambda\sigma_z^2}{\sigma_\varepsilon^2 + \sigma_{\psi,\psi}^2}\), and with \(k^* \in (k, \bar{k})\). Furthermore, there exists a unique fraction \(\alpha^* \in (0, 1)\) of informed investors in equilibrium iff the combination of parameters is such that \(k \in (k^*, \bar{k})\).

Proof: See the Appendix.

The above result proves that an overall equilibrium exists in this market, and it is unique, if and only if the combination of parameters belong to the subspace that is completely characterized by the cost of acquiring information \(k\). Particularly, if \(k \leq k^*\), acquiring information is so cheap that everyone would want to do it, and so this cannot constitute an equilibrium. On the other hand, for \(k \geq \bar{k}\), acquiring information is prohibitively expensive.
and no one would be willing to incur such a cost, which again cannot be sustained as an equilibrium.

The overall equilibrium, when it exists, is completely determined by equation (8) (for $\alpha^*$ in lemma 2), and the equations for the coefficients that determine the equilibrium in trading activity in lemma 1. From now on, we restrict attention to this subspace of parameters for which the equilibrium exists to study how the information contained in the price is affected.

Having encountered an analytical expression for $\alpha^*$, and shown the conditions that ensure the existence and uniqueness of this equilibrium, we now proceed to characterize how the fraction of informed investors depends on the prior uncertainty about the fundamentals of the firm ($\sigma_v^2$) and the intensity with which (incorrect) uninformed investors react to the salient information of a change in rating ($\lambda$), both parameters directly linked to the effects of change in credit rating.

**Lemma 3 (The effect of $\lambda$ on $\alpha^*$.)** In the subspace of parameters for which an overall equilibrium exists, the fraction of informed investors $\alpha^*$ is decreasing in $\sigma_v^2$ when $k$ and $\lambda$ are both simultaneously low enough.

Proof: See Appendix.

This last result shows that net benefits of becoming informed in equilibrium are not always increasing in the prior uncertainty about the risky asset. As it is the case in Grossman and Stiglitz (1980), higher uncertainty about the asset implies a larger benefit of privately acquiring information about it, but it also implies larger benefits of remaining uninformed as the price becomes more informed.

When the cost of acquiring information is extremely low, the fraction of informed investors will tend to be high. Under this scenario, a marginal increase in the prior uncertainty would generate relatively small incentives to become informed. Since in this scenario the price is already highly informative ($k$ very low), what an uninformed individual learns about the risky asset can be enough to profit from trading against noise trading that reacts to the change in rating. Overall, we could see a lower $\alpha^*$ if the latter benefit (for uninformed agents) dominates, and no other benefits of becoming informed are present to prevent it. Particularly, that is the case when the probability $\lambda$ of reacting naively for any uninformed
agent is low enough. When such a risk (i.e., $\lambda$) is larger, the extra benefit of avoiding potentially trading on incorrect valuations becomes significant enough to overturn the effect of a low cost of information acquisition, thus leading to an increase in $\alpha^*$. This result is an important step towards understanding the effects at play in our next analysis on how informative (or, analogously, noisier) the equilibrium stock price of the firm becomes after an upgrade or downgrade in its corresponding credit rating.

With the existence and uniqueness of an overall equilibrium being proved, our next goal is to analyze how the equilibrium price informativeness is affected by a change in rating, particularly focusing on a potential difference between the effect of an upgrade versus a downgrade, relative to our benchmark case with no change in rating. As already stated, we follow Kyle (1985b) in measuring the informativeness of the stock price by computing the extra noise that is perceived about the fundamental value after filtering out all the relevant information contained in the equilibrium price, $Var[\tilde{v}|P]$. Thus, a higher value of this measure will be directly associated with a relatively noisier price or, more importantly for our interpretations, a less informative price (from an uninformed agent’s perspective). Computing the general formula for such conditional variance, and replacing the coefficients obtained for the equilibrium in trading activity, we obtain the following expression for our measure of price informativeness:

$$Var[\tilde{v}|P] = \sigma_v^2 - \frac{\sigma^4_v}{\left(\sigma^2_v + \frac{\gamma^2}{\beta^2} \sigma^2_\epsilon + \frac{\eta^2}{\beta^2} \sigma^2_\zeta\right)} = \sigma_v^2 \left[\frac{c^2 \sigma^2_\zeta + (1 - \alpha)^2 \lambda^2 \sigma^2_\zeta}{\alpha^2 \sigma^2_\epsilon + c^2 \sigma^2_\zeta + (1 - \alpha)^2 \lambda^2 \sigma^2_\zeta}\right], \quad (9)$$

for a generic $\sigma^2_v$, and where $\alpha$ must be replaced by the corresponding expression describing the fraction of informed investors in equilibrium.

Before any analysis of our measure of price informativeness, there are a few details about the model important to keep in mind. First, it must be noted that all informational effects are triggered by the change in the rating. We are modeling a situation in which all investors acknowledge that a change in rating will be happening with probability one in the near future, with all the consequences that such announcement has, including the existence of incorrect uninformed investors. For such a situation of a change in rating happening with certainty tomorrow (and with $\lambda \in (0, 1)$), we are now interested in studying how
price informativeness changes when the change is an upgrade compared to the case of a
downgrade.

It is also important to recall that the parameter that determines whether the scenario
under consideration corresponds to an upgrade or a downgrade is the rating $\psi$ or, more
specifically, whether the precision of the prior about the value of the fundamental goes up
or down. Depending on the direction (or sign) of this change, the equilibrium is completely
characterized by the equations provided above. Keeping this in mind, for a given probability
of estimating a wrong valuation if staying uninformed ($\lambda$), we can study the change in
$Var[\tilde{v}|P]$ as we let a generic $\sigma_v^2$ vary up or down, accounting for the implied change in $\lambda$
according to our model. In particular, a downgrade (upgrade) is associated with larger
(smaller) values of $\sigma_v^2$. At the same time, according to our modeling of limited attention,
$\lambda > 0$ only for the case of a downgrade (when the variance of the risky asset is high enough).

This leads to our next proposition:

**Proposition 2 (Differential effect of a change in credit rating on stock price informativeness)**

*In equilibrium, after a downgrade in the credit rating of the firm, the stock price becomes
strictly more informative. After an upgrade, the informativeness of the stock price does not
change.*

Proof: See the Appendix.

The result is a consequence of the cost-benefit analysis performed by investors to decide
whether to become informed at $t = 0$, as discussed above, together with the resulting
environment in which those informed agents can trade more or less aggressively after a
change in rating. The effects of a change in the rating can be separated into two: the
change in the prior uncertainty about the fundamentals ($\sigma_v^2$), and the consequent effect on
the probability of suffering the consequences of limited attention. In the proof we show that
stock price informativeness in equilibrium is monotonically increasing in both, $\sigma_v^2$ and $\lambda$.

To understand the intuition behind the result, we can first consider the marginal costs
and benefits of becoming informed provided by $\lambda$. A downgrade involves a greater compo-
nent of trading activity purely uninformative (besides the standard noise trading) coming
from the incorrect uninformed investors. This creates a benefit of acquiring information
since informed trades will be less revealing through price movements. Such a benefit is further amplified when an investor considers the alternative of remaining uninformed. That option involves a greater relative cost linked to the risk of trading naively at the time of the change in rating. In equilibrium, a larger $\lambda$ initially introduces noise (trading) into the price system, but it also creates benefits of becoming informed that lead to a price marginally more informative in the end.

The effect of $\sigma_v^2$ itself is more standard in the literature, and works as follows. Larger prior uncertainty about $\hat{v}$ implies a greater advantage for those that become informed. As this marginally leads to more investors becoming informed, the price would tend to reveal more information, which generates a benefit of staying uninformed. If no incorrect reaction from some traders were possible in the model (the role of $\lambda > 0$), these two effects would cancel each other in equilibrium, keeping price informativeness invariant. The existence of the incorrect reaction from some uninformed traders is what generates the (amplified) added benefit of becoming informed that implies a positive net effect of larger $\sigma_v^2$ on price informativeness in equilibrium. As mentioned, both effects lead to an increase in price informativeness, which directly proves that a downgrade will have this effect on the stock price.

On the other hand, for an upgrade, $\sigma_v^2$ is reduced, but $\lambda = 0$ in this case. As it is mentioned above, shutting down the effect of $\lambda$ makes the effect of $\sigma_v^2$ be perfectly anticipated by all rational investors in the economy. Thus, the reaction of investors that decide to become informed (going down relative to the initial state) cancels out with the reduction in noise associated to the lower $\sigma_v^2$. In equilibrium, price informativeness remains the same. This result is standard in the literature, originally stated in Grossman and Stiglitz (1980).

This last result motivates our main empirical prediction:

**Prediction 1:** The stock price is more informative following a credit rating downgrade.

**Prediction 2:** The stock price does not become significantly more or less informative following a credit rating upgrade.
3.3 Further Cross-Sectional Predictions From The Model

3.3.1 A partially anticipated change in rating

So far, we have assumed that the change in rating is anticipated by all investors in the market with probability one. This is a simplification of the model that we argue would not change its resulting main predictions. It would just have an affect on the magnitudes of each of the effects already described.

particularly, if we introduce a commonly perceived probability $h$ that the rating remains unchanged, while with probability $(1 - h)$ the corresponding change in rating is expected, investors would face a binomial tree at $t = 0$, when deciding whether to become informed or not. The branch of the tree associated with the change in rating corresponds exactly to the equilibrium that we already described here.

The second branch, without a change in rating, particularly involves a middle level of the probability of an incorrect reaction from some of the uninformed traders, which is the benchmark situation against which we compare the effects of a change in rating. It also keeps the prior variance about the fundamentals at the original level. This means that the informativeness of the stock price (and the fraction of informed investors in equilibrium) will be at the benchmark of our current model.

Since in this modified model agents would not know which of the two scenarios they will be facing at $t = 0$, the resulting fraction of informed agents is a weighted average of the corresponding fractions in each of the two cases, hence an intermediate level between the benchmark and the value that we consider in our model with $h = 0$. This in turn would lead to all the other effects in place in the main model to adapt in proportional magnitudes given the rationality assumed for all investors at the time they decide to become informed. Importantly, all the effects described in the main model would be still observed in this case, in the same direction as before.

Arguably, whenever changes in rating occur in practice they are highly anticipated. This would be consistent with a probability $h$ close to zero in the modified model described above. Nonetheless, it seems intuitive to think that, all else equal, some firms (or their bonds) may be perceived as having a larger probability $h$ than others, for example based
on their current credit rating. Hence, the discussion presented in this subsection provides us with a further prediction that results from the model.

**Prediction 3:** Predictions 1 and 2 still hold (though increasingly weakened) as the probability that the rating remains unchanged gets strictly greater than zero.

### 4 Sample Selection and Construction

We collect data from four databases. First, we obtain information on credit ratings from Compustat. Our measure of credit rating is the monthly S&P Long-Term Issuer Level rating obtained from the ADSPRATE database. We convert the letter ratings into numerical equivalents using an ordinal scale ranging from 1 for the highest rated firm (AAA) to 22 for the lowest rated (D). Second, we obtain intra day transaction data from the Trade and Quote (TAQ) database. This database contains the daily trades and quotes. We infer the sign of each trade using the Lee and Ready (1991) algorithm. We follow previous studies and employ a “zero” minute rule for trades executed after 2000. Third, we obtain firm’s stock prices and return information from the Center for Research in Security Prices (CRSP). We compute the industry return as the three-sic digit weighted average return. Fourth, we collect information on ownership by institutions and their characteristics from Thomson Reuters.

We start with the monthly S&P Long-Term Issuer Level dataset and identify all the credit rating downgrades and upgrades that occurred between January 2000 and December 2013. We restrict our attention to the firms whose stocks were traded on the NYSE, AMEX and NASDAQ at the time of the credit rating change. We retain the rating changes for which there is complete financial information on the Compustat-quarterly tapes for the issuer. This procedure results in 2,671 rating downgrades and 1,709 rating upgrades. For each of these each rating changes, we construct a 2 quarters before and 4 quarters after panel around the event.

We now describe the construction of the measure of stock price informativeness.
4.1 Measures of Stock Price Informativeness

We employ four different measures to gauge the informativeness of stock prices. The first two measures are market-based while the other two are computed based on the ownership structure of the issuer.

4.1.1 Return Non Synchronicity

Roll (1988) found that firm-specific stock price movements are generally not correlated with identifiable news release, suggesting therefore that private information is especially important in the explanation of firm specific stock price movements. His argument is as follows. Prices move upon new information and such new information is incorporated into prices in two different ways. First, through identifiable news such as earnings release or other public information. Second, through the trading activity of speculators who trade on private information. The extent of price movements not correlated with public news and related to trading activity of speculators or informed trading constitutes our first measure. We follow Chen et al. (2007) and measure the extent of firm-specific price movements or price nonsynchronicity by estimating the $1 - R^2$, where $R^2$ is the R-square from the following regression:

\[ r_{i,j,t} = \beta_{i,0} + \beta_{i,m} r_{m,t} + \beta_{i,j} r_{j,t} + \epsilon_{i,t} \] (10)

Here, $r_{i,j,t}$ is the return of firm $i$ in industry $j$ at time $t$, $r_{m,t}$ is the market return at time $t$, and $r_{j,t}$ is the return of three-digit SIC industry $j$ at time $t$. We estimate the return nonsynchronicity per quarter. A larger nonsynchronicity is expected to be positively correlated with the amount of a stock’s private information. While this first measure of private information may capture other sources of stock return variations unrelated to private information, hence introducing some noise in the measure, previous studies show that price nonsynchronicity reflects proportionally more private information than noise (see for example Durnev et al. (2003), Durnev et al. (2004)).\(^\text{17}\) In sum, we expect the return \textit{Non Synchronicity} to be positively related with the amount of informed trading and stock price.

\(^{17}\)Chen et al. (2007) provide an extensive list of references that find a positive relationship between firm-specific information and return nonsynchronicity.
4.1.2 Probability of Informed Trading

Our second measure of informed trading is the Probability of Informed Trading (PIN). The PIN measure has strong theoretical foundation as a measure of the amount of private information in stock prices. The measure was first developed by Easley et al. (2002) and it is based on a structural market microstructure model in which trades come from uninformed and uniformed traders. The PIN measure gauges the probability of informed trading in a stock, thus it is conceptually a sound measure for the amount of private information reflected in stock prices. The intuition behind the measure is as follows. A competitive market maker trades with uninformed and informed traders. Trade occurs over $T$ discrete trading days, and within each day, trading occurs in continuous time. Information events occur between trading days with probability $\alpha$. When these events are realized, these events are either bad news with probability $\delta$ or good news with probability $1-\delta$. Traders informed of good news buy and traders informed of bad news sell. By assumption, the orders from these informed traders follow a Poisson process with a daily arrival rate $\mu$. On the other hand, uninformed traders trade for liquidity reasons. It is assumed that buy and sell orders from uninformed traders each arrive at the market according to a Poisson process with a daily arrival rate of $\epsilon$. The likelihood function for a single trading day is given by:

$$L(\theta|B,S) = \alpha(1-\delta)e^{-\mu-2\epsilon}B^SE^{\mu+\epsilon}B!S! + \alpha\delta e^{(-\mu-\epsilon)}B^S(\mu+\epsilon)^SE^{B+S}B!S! + (1-\alpha)e^{(-2\epsilon)}B^SE^{B+S}B!S!$$  \hspace{1cm} (11)

This particular factorization of the likelihood function appears in Easley et al. (2008). Here $B$ is the number of buy orders and $S$ is the number of sell orders in a trading day. Using trading information over $Q$ days and assuming cross-trading-day independence, one can estimate the set of parameters $\{\alpha, \delta, \mu, \epsilon\}$ by maximizing the following likelihood function:
\[ P = \prod_{i=1}^{Q} L(\theta | B_i, S_i) \] (12)

For the exact closed form solution of the log-likelihood function, please refer to equation (2) of Easley et al. (2008). Then, the probability of informed trading in a given stock for a given period of trading days \( Q \) is:

\[ PIN = \frac{\alpha \mu}{\alpha \mu + 2\epsilon} \] (13)

The PIN measure is expected to be low for stocks with less fluctuations of daily buy and sell orders. If these two are roughly balanced, then these orders are more likely from investors’ independent liquidity needs. The law of large numbers smooths out these orders and the probability of information events, \( \alpha \), is small. Analogously, the measure of informed trading is expected to be larger for stocks with frequent buy and sell orders that deviate from the normal order flow. The PIN measure has been used in previous studies. Easley et al. (2002) related the PIN measure to the asset pricing literature and found that the risk of private information (as captured by this measure) is priced, so that high PIN stocks earn higher expected returns. More recently, Vega (2006) found further evidence supporting PIN as a measure of private information in price. She showed that stocks with higher PIN values have smaller post-earnings-announcement drifts, suggesting high PIN stocks adjust to fundamentals quicker. As she found that this quicker adjustment is not because of media coverage or other public news releases, her results suggest that stocks with high PIN values likely contain more private information by speculators. In sum, we expect the PIN measure to be positively related with the amount of informed trading and stock price informativeness.

4.1.3 Active Mutual Fund Ownership

One concern with the return Non Synchronicity and PIN measure is the fact that these may sometimes fail to capture informed trading. For instance, Collin-Dufresne and Fos (2015) find that activist investors, which are likely to be more informed, trade on days where the stock liquidity is high, thereby leading to a negative relationship between measures of adverse selection and informed trading. To check whether this is a concern in our sample,
as a robustness check, we compute two additional ownership-based measures.

Using quarterly fund holdings from Thomson Reuters, we compute the quarterly holdings for U.S. equity mutual funds during the period from 2000 to 2013. We exclude all ownership by international funds, municipal bond funds, “bond and preferred” funds, sector funds, and index funds to focus on actively managed domestic equity funds. Our third measure of stock price informativeness is:

\[
\text{Active Mutual Fund Ownership}_{it} = \frac{\sum_{k=1}^{K} \text{Fund Own}_{kit}}{\text{Institutional Ownership}}
\]

where \(\text{Fund Own}_{kit}\) is the ownership of the active mutual fund \(k\) in firm \(i\) at time \(t\). We normalize the mutual fund ownership by the total institutional ownership to account for cross-sectional differences in the level of ownership by institutions.

4.1.4 Long-Term Institution Ownership

Our last measure is motivated by the findings of Gaspar, Massa, and Matos (2005), Derrien, Kecskés, and Thesmar (2013) that argue that the benefits of gathering information about firms are higher and that the use of such information is more efficient for firms with a longer investment horizon.\(^{18}\) To the extent that there are fixed costs of information acquisition, the net benefit of information acquisition increases with the time an investor holds a firm in its portfolio.

We follow the aforementioned studies to compute the ownership of long-term institutions. First, we start at each institutional investor level and measure the horizons based on the portfolio churn rate. Let \(Q\) denote the set of companies held by institution \(i\). The quarterly churn rate of institution \(i\) is computed as:

\[
CR_{it} = \frac{\sum_{j \in Q} \|N_{jit} * P_{jt,t} - N_{jit-1} * P_{jt-1,t} - N_{j,t-1} * \Delta * P_{jt,t}\|}{\sum_{j \in Q} N_{jit+P_{jt,t}+N_{j,t-1}+P_{jt-1}}^{2}}
\]

where \(N_{jit}\) and \(P_{jt,t}\) represent the number of shares and the stock price of company \(j\) held by institution \(i\) at the end of the quarter \(t\). Second, we average the churn rate over the last four quarters. We then classify an institution as a long term institution if the average portfolio

\(^{18}\)This, in terms of influencing managerial behavior (see also Harford, Kecskes, and Mansi (2016)).
turnover rate is in the bottom 33% of the distribution of churn rate for each quarter \( t \). Finally, we compute our measure of long term institution ownership as:

\[
\text{Long Term Institution Ownership}_{it} = \frac{\sum_{h \in L} \text{Ownership}_{iht}}{\text{Institutional Ownership}_{it}}
\]  

(14)

where \( h \) denotes the set of institutional investors classified as long term holding shares of issuer \( i \). We normalize the above measure by the total institutional ownership for issuer \( i \) to account for cross sectional differences in the levels ownership by institutions.

4.2 Matched Samples

We test our predictions by comparing the stock price informativeness of firms that experienced a rating change to a matched set firms. The construction of the matched sample is especially complex given the myriad outcome variables we model. We overcome this by constructing a matched sample for each proxy measure of stock price informativeness.

For each outcome variable, we construct the control sample by matching firms that experienced a rating change to U.S public firms on rating, year and month and Altman Z-score. Specifically, for every outcome variable, for every firm that experienced a rating change, we identify one unique control firm that is the closest to the firm experiencing the rating change in terms of the Altman Z-score, current and lagged values of the outcome variable in the quarter prior to the rating change. We also require the matched firm to have the same rating in the quarter before the rating change. We refer to as control sample CS1, CS2, CS3, and CS4 the control sample where we also match on Non-Synchronicity, PIN, Active Mutual Fund Ownership and Long Term Institution Ownership, respectively.

5 Empirical Methodology

In our empirical tests, we a use difference-in-differences methodology and compare the measures of stock price informativeness for the firms that experienced a rating change with the stock price informativeness for matched firms around the event date. Specifically, we estimate the following fully saturated specification with quarter -1 (the quarter before the event when the matching is performed) as the base period:
\[ y_{it} = \beta_0 + \sum_{t=0}^{4} \lambda_t \ast \delta_t \ast Treated_i + \sum_{t=0}^{4} \theta_t \ast \delta_t + \mu_1 \ast \delta_{-2} + \mu_2 \ast \delta_{-2} \ast Treated_i + \phi \ast Treated_i \\
+ \Delta'X_{it-1} + \alpha_i + \gamma_t + q_t + \epsilon_{it} \]

(15)

where \( y_{it} \) is one of the measures of stock price informativeness for issuer \( i \) at time \( t \). \( Treated_{it} \) is an indicator variable that takes the value of one if the firm experienced a rating change, and it is zero otherwise. \( \delta_k \) is an indicator variable that takes the value of one if it is \( k \) quarters after the event, and it is zero otherwise. In our tests we focus on a 6 quarter window that goes from two quarters before the event to 12 quarters after that. The model is fully saturated with the quarter prior to the rating change as the excluded category. \( X_{it-1} \) is a set of lagged firm-specific time varying controls. We follow Gompers et al. (2003) and Yan and Zhang (2009) and include a set of control variables that account for the demand of the stock by institutions. Such controls include the issuer’s Size, Share Turnover, Dividend Yield, Market-to-Book, Stock Return and Stock Price. \( \gamma_t \) and \( q_t \) are year and quarter fixed effects, respectively. \( \alpha_i \) is a firm-level fixed effects. Since \( \alpha_i \) is perfectly collinear with \( Treated_{it} \), the treatment dummy cannot be identified and so is not reported. Standard errors are clustered at the firm level. The model is fully saturated with the quarter immediately before the change in the rating as the base period (i.e., quarter -1). Thus, the coefficients \( Treated_{i} \ast \delta_{t} \) estimate the change in the stock price informativeness in the treated firms relative to control firms’ “t” quarters after the rating change date as compared to quarter -1.

One important assumption in our empirical methodology is that there is no differential change in the measures of stock price informativeness for the firms that experienced a rating change relative to the set of control firms before the downgrade/upgrade. To test this, in Figures 1 and 2 we plot the quarterly values of the outcome variables for the firms experiencing a rating change and the set of control firms during the three quarters prior to the event to four quarters after. The graphs also provide a 90% confidence interval along with the mean values. Focusing on Figure 1, the average values of our outcome variables for the firms that experienced a rating downgrade and the set of matched firms
prior to the event is not statistically different for any of the pre-event quarters. Figure 2 plots the average value of the outcome variables for the firms experiencing a rating upgrade and the set of control firms three quarters before the event to four quarters after. Once again, the mean values for firms that experienced a rating upgrade and the set of control firms is statistically similar, thus validating the parallel trend assumption and the matching procedure.

5.1 Summary Statistics

Table 1 presents the distribution of rating changes per year and one-digit SIC code of the issuer. Out of the 2,671 rating downgrades, 1,604 are single-notch downgrades. The number of rating downgrades is also clustered around the year 2000 and the year 2008. This is not surprising as this related to the business cycle and economic downturns. In terms of the rating upgrades, out of the 1,709 rating upgrades, 1,510 are single-notch upgrades. The number of rating upgrades is clustered around the year 2010, which again, coincides with the beginning of the expansion of the business cycle. Panel B presents the number of rating changes based on the issuer’s industry. A large proportion of our sample constitutes downgrades or upgrades of manufacturing firms (SIC codes (2) and (3)) followed by transportation and utilities (SIC code (4)).

Tables 2 and 3 present the summary statistics for all single-notch multiple-notch credit rating downgrades and upgrades at the time of the matching. Once again, we find that the value of the proxies of stock price informativeness is larger for issuers that experienced a credit rating downgrade.

6 Results

6.1 Univariate Results

In Figure 1 we plot the mean value of market-based and ownership based measures of stock price informativeness for the firms that experienced a downgrade and control firms. Panel A, left panel, reports the mean values for Non-Synchronicity; there is no statistical difference between treated and control firms prior to the downgrade. However, the Non-Synchronicity
for the treated firms diverges from that of the control firms in the quarter of the rating downgrade. The mean values are statistically similar from quarter 2 onwards. Panel A, right figure reports the mean value of PIN. While there is no statistical difference in the PIN measure for firms that experienced a rating downgrade and the set of control firms in any of the two quarters prior to the event date, the PIN measure is larger for the treated firms in the quarter of the event and the quarter immediately after.

Figure 1, panel B depicts the average value for the ownership-based measures of price informativeness for firms that experienced a downgrade and the set of control firms. While the average values of Mutual Fund Ownership and Long Term Institution Ownership is statistically similar for treated and control firms prior to the event, the Mutual Fund Ownership and Long Term Institution Ownership diverges for the treated firm from that for the control firms in the quarter of the event and the quarter following the downgrade, respectively. Overall, these findings are consistent with Prediction 1.

Figure 2 depicts the average values of the four measures of stock price informativeness for firms that experienced a rating upgrade and the set of control firms. Consistent with Prediction 2, we do not find a statistically difference in the average value of the proxies for treated and control firms.

### 6.2 Multivariate Results

Table 4 presents the results of the regression estimating the change in market-based measures of stock price informativeness following a credit rating downgrade. The sample in columns (1), (2), (5), and (6) is CS1 while the sample in columns (3), (4), (7), and (8) is CS2 (please refer to Section 4.2 for a detailed description of the construction of such matched samples). The omitted category in these tests is the quarter before the treatment. Thus, the coefficients on the interaction terms represent the extent to which stock price informativeness is different between the treated and control firms in that quarter relative to one quarter before the downgrade.

Columns (1)-(4) includes all the firms that experienced a credit rating downgrade and the respective set of controls. In columns (1) and (2) the dependent variable is the return Non-Synchronicity and the matched sample is CS1. Column (1) presents the results where
we just control for year, quarter and firm level fixed effects. The insignificant coefficient on $Treated \times \delta_{-2}$ indicates that the Non-Synchronicity for the treated firms is not different from that of the control firms in the two quarters before the downgrade. This again confirms the lack of pre-trends in the data. The positive and significant coefficient on $Treated \times \delta_0$ dummy variable indicates that in the quarter of the downgrade, the Non-Synchronicity is higher for the firms that experience a rating downgrade. For the median value of Non-Synchronicity of 0.580, this represents a 3.8% increase relative to control firms. Moreover, we find the difference to be persistent over two, three and four quarters after the downgrade. The increase in the Non-Synchronicity measure is consistent with the prediction that the stock price informativeness increases following a rating downgrade. In column (2) we include a set of controls to account for cross sectional heterogeneity. We again find a positive and significant coefficients on $Treated \times \delta_0$, $Treated \times \delta_1$, $Treated \times \delta_2$, $Treated \times \delta_3$, and $Treated \times \delta_4$. In addition, the magnitude of the coefficient is similar to the one in column (1). We find that smaller firms, with higher stock price have lower stock price informativeness.

In columns (3) and (4) the dependent variable is PIN and the matched sample is CS2. Columns (3) reports the results including year, quarter and firm-level fixed effects. The coefficient on $Treated \times \delta_{-2}$ indicates that the average value of PIN for the treated firms is similar to that of the control firms. Once again, we find a positive and significant coefficient on $Treated \times \delta_0$. For the median value of PIN for firms that experienced a rating downgrade of 0.117, this represents a 4.2% increase in the PIN measure.

Columns (5)-(8) include only the set of firm that experienced multiple-notch downgrades along with the set of matched firms and repeat our tests on this subsample. We again find that the market-based measures of stock price informativeness are higher for the firms that experience a multiple notch downgrade relative to the set of control firms. Interestingly, the magnitude of the coefficient on $Treated \times \delta_1$ is almost the double than the one reported in columns (1)-(4). Once again, this is consistent with Prediction 1.

In Table 5 we repeat our tests using the set of ownership-based measures of stock price informativeness. The dependent variable in columns (1) and (2) is Active Mutual Fund Ownership and the control sample is CS3. We do not find a statistically significant difference in the average values of Active Mutual Fund Ownership between treated and control firms prior and after the event. In columns (3) and (4), the dependent variable
is Long Term Institutional Ownership and the sample is the matched sample CS4. Once again, we do not find a statistically significant difference in the average values of Long Term Institutional Ownership. In columns (5)-(8) we repeat our tests using the ownership-based measures of stock price informativeness on a subsample of firms that experienced multiple-notch downgrades. Columns (5) and (6) present the results using Active Mutual Fund Ownership as dependent variable; the control sample is CS1. The coefficient on Treated*δ0 on columns (5) and (6) indicate that the share of Active Mutual Fund Ownership is higher for the treated firms at the time of the multiple-notch downgrade relative to the set of control firms. For the median value of Active Mutual Fund Ownership for firms that experienced a multiple notch-downgrade of 0.182, this represents a 25% increase in the share of the total institutional ownership that is owned by Active Mutual Funds. In columns (7) and (8) we report the estimates using Long Term Institutional Ownership as a measure of stock price informativeness. The matched sample is CS4 and the sample consists of the firms that experienced a multiple-notch downgrade along with the set of matched firms. The coefficient on Treated * δ1 on columns (7) and (8) indicates that the share of the institutional ownership that is held by Long Term Institutions is larger in the quarter after the multiple-notch downgrade relative to the set of matched firms. The magnitude is also economically significant. For the median firm, this represents a 9.5% (0.019/0.199) increase. Once again, these results are consistent with Prediction 1.

Tables 6 repeats our tests focusing on the set of firms that experienced a credit rating upgrade along with the set of control firms when using the market-based measures of stock price informativeness. We do not find a statistical difference in the average value of the proxies of stock price informativeness between treated and control firms. Table 7 replicates the specification of Table 6 when using the set of ownership-based measures of stock price informativeness. Once again, we do not find a statistical difference in the value of the outcome variables between treated and control firms after the rating upgrade. The statistically insignificant coefficient on Treated * δ0, Treated * δ1, Treated * δ2, Treated * δ3, and Treated * δ4, is partially consistent with Prediction 2.
7 Conclusion

In this paper we study the specific effect that a change in rating has on the informativeness of the stock price. Particularly, our theoretical model predicts that after a downgrade price informativeness increases and, after an upgrade, prices informativeness does not change. We find empirical support for those theoretical predictions.

Besides the characterization of the effects of a change in rating on the informativeness of the stock price, in our theoretical work we have shown how adding the possibility of some unsophisticated investors that face the consequences of having limited attention affects the outcome in a noisy rational expectations model. Without this feature of the model, any potential change in the composition of investors acquiring information would be rationally anticipated leaving the informativeness of the stock price unaltered. When rational investors consider the possibility of them not being able to allocate enough attention to correctly estimate the value of the risky asset if they remained uninformed, the decision of becoming informed endogenously affect the amount of noise trading linked to the change in rating. This leaves room for some of the effects of a change in rating to be not fully anticipated thus allowing for a non-zero effect on price informativeness in equilibrium.

We find empirical support for the predictions of our model that considers traders with some limitation in their capacity to learn from prices at all times. This may be interpreted as evidence of the presence and effects of limited attention in at least part of the population of investors in the market, interacting with some more sophisticated investors with better access to information processing technologies. It seems interesting to further study if this type of effect on stock price informativeness arises in the presence of some other events that could affect investors’ limited attention.

Public releases of information about a firm that are widely anticipated and internalized by sophisticated investors, affect how informative the stock price is about the fundamentals. We have shown that this can happen not just because of the particular information provided by the public announcement, but because of the related effect of this change in the public informational environment. More or less noise in public information about a risky asset affects the likelihood that some less sophisticated investors may not be able to allocate enough attention to learning about the asset when it is most needed. This joint effect of
changes in the available information and the risk of suffering the consequences of limited attention ends up determining the incentives to become informed and to trade on that private information.
Appendix A: Figures and Tables

Figure 1: Credit Rating Downgrades - Parallel Trend Assumption

Panel A: Market-Based Measures of Price Informativeness

Panel B: Ownership-Based Measures of Price Informativeness
Figure 2: Credit Rating Upgrades - Parallel Trend Assumption

Panel A: Market-Based Measures of Price Informativeness

Panel B: Ownership-Based Measures of Price Informativeness
Table 1: Distribution of Downgrades and Upgrades

Table 1, Panel A presents the number of credit rating downgrades and credit rating upgrades per year, from 2000 to 2013. Table 1, Panel B presents the number of credit rating downgrades and credit rating upgrades based on the issuer’s one-digit SIC code. The sample includes all the firms whose stocks was traded at the NYSE, AMEX, and NASDAQ at the time of the credit rating change and for which there is complete information for the set of control variables.

Panel A: Distribution of Rating Changes per Year

<table>
<thead>
<tr>
<th>Year</th>
<th>Single Notch Downgrades</th>
<th>Multiple Notch Downgrades</th>
<th>Total Downgrades</th>
<th>Single Notch Upgrades</th>
<th>Multiple Notch Upgrades</th>
<th>Total Upgrades</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>109</td>
<td>111</td>
<td>220</td>
<td>58</td>
<td>13</td>
<td>71</td>
</tr>
<tr>
<td>2001</td>
<td>157</td>
<td>123</td>
<td>280</td>
<td>58</td>
<td>19</td>
<td>77</td>
</tr>
<tr>
<td>2002</td>
<td>176</td>
<td>77</td>
<td>253</td>
<td>47</td>
<td>6</td>
<td>53</td>
</tr>
<tr>
<td>2003</td>
<td>114</td>
<td>56</td>
<td>170</td>
<td>89</td>
<td>13</td>
<td>102</td>
</tr>
<tr>
<td>2004</td>
<td>94</td>
<td>74</td>
<td>168</td>
<td>98</td>
<td>14</td>
<td>112</td>
</tr>
<tr>
<td>2005</td>
<td>104</td>
<td>76</td>
<td>180</td>
<td>126</td>
<td>21</td>
<td>147</td>
</tr>
<tr>
<td>2006</td>
<td>101</td>
<td>89</td>
<td>190</td>
<td>126</td>
<td>11</td>
<td>137</td>
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<tr>
<td>2007</td>
<td>122</td>
<td>89</td>
<td>211</td>
<td>135</td>
<td>15</td>
<td>150</td>
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<tr>
<td>2008</td>
<td>222</td>
<td>110</td>
<td>332</td>
<td>87</td>
<td>9</td>
<td>96</td>
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<tr>
<td>2009</td>
<td>94</td>
<td>57</td>
<td>151</td>
<td>101</td>
<td>15</td>
<td>116</td>
</tr>
<tr>
<td>2010</td>
<td>58</td>
<td>45</td>
<td>103</td>
<td>175</td>
<td>30</td>
<td>205</td>
</tr>
<tr>
<td>2011</td>
<td>101</td>
<td>54</td>
<td>155</td>
<td>162</td>
<td>12</td>
<td>174</td>
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<tr>
<td>2012</td>
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<td>55</td>
<td>133</td>
<td>89</td>
<td>6</td>
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</tr>
<tr>
<td>2013</td>
<td>74</td>
<td>51</td>
<td>125</td>
<td>159</td>
<td>15</td>
<td>174</td>
</tr>
</tbody>
</table>

1604 | 1067 | 2671 | 1510 | 199 | 1709 |

Panel B: Distribution of Rating Changes per Industry

<table>
<thead>
<tr>
<th>SIC Code</th>
<th>Single Notch Downgrades</th>
<th>Multiple Notch Downgrades</th>
<th>Total Downgrades</th>
<th>Single Notch Upgrades</th>
<th>Multiple Notch Upgrades</th>
<th>Total Upgrades</th>
</tr>
</thead>
<tbody>
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<td>(0)</td>
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<td>1</td>
<td>8</td>
<td>6</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
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<td>81</td>
<td>213</td>
<td>151</td>
<td>20</td>
<td>171</td>
</tr>
<tr>
<td>(2)- (3)</td>
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<td>444</td>
<td>1084</td>
<td>594</td>
<td>72</td>
<td>666</td>
</tr>
<tr>
<td>(4)</td>
<td>294</td>
<td>170</td>
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<td>(5)</td>
<td>167</td>
<td>98</td>
<td>265</td>
<td>164</td>
<td>7</td>
<td>171</td>
</tr>
<tr>
<td>(6)</td>
<td>205</td>
<td>114</td>
<td>319</td>
<td>174</td>
<td>26</td>
<td>200</td>
</tr>
<tr>
<td>(7) - (8)</td>
<td>153</td>
<td>156</td>
<td>309</td>
<td>188</td>
<td>16</td>
<td>204</td>
</tr>
<tr>
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<td>6</td>
<td>3</td>
<td>9</td>
<td>0</td>
<td>0</td>
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</tr>
</tbody>
</table>

Total | 1604 | 1067 | 2671 | 1510 | 199 | 1709 |
Table 2 presents the summary statistics in the quarter prior to the credit rating change for single notch upgrades/downgrades. The sample includes all the rating changes for which there is complete information for the set of control variables.

<table>
<thead>
<tr>
<th></th>
<th>All Rating Changes</th>
<th>Rating Downgrades</th>
<th>Rating Upgrades</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N</td>
<td>p25</td>
<td>p50</td>
</tr>
<tr>
<td>Non Synchronicity</td>
<td>3,114</td>
<td>0.331</td>
<td>0.543</td>
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<tr>
<td>PIN</td>
<td>2,373</td>
<td>0.075</td>
<td>0.106</td>
</tr>
<tr>
<td>Institutional Ownership</td>
<td>2,724</td>
<td>0.542</td>
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<td>Active Mutual Fund Ownership</td>
<td>2,631</td>
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<td>Long Term Institutions Ownership</td>
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<td>0.139</td>
<td>0.203</td>
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<tr>
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<td>12</td>
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<tr>
<td>Investment Grade</td>
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<td>1.000</td>
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<tr>
<td>Altman Z-Score</td>
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<td>411.641</td>
<td>1,056.105</td>
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<tr>
<td>Dividend Yield</td>
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<td>0.003</td>
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<td>Return Volatility</td>
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<td>Log(Price)</td>
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<td>Market-to-Book</td>
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<td>Stock Return</td>
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Table 3: Summary Statistics - Multiple Notch Rating Change

Table 3 presents the summary statistics in the quarter prior to the credit rating change for multiple notch upgrades/downgrades. The sample includes all the rating changes for which there is complete information for the set of control variables.

<table>
<thead>
<tr>
<th></th>
<th>All Rating Changes</th>
<th>Rating Downgrades</th>
<th>Rating Upgrades</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N</td>
<td>p25</td>
<td>p50</td>
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<tr>
<td>Non Synchronicity</td>
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<td>PIN</td>
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<td>Active Mutual Fund Own.</td>
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<td>1,266</td>
<td>-0.114</td>
<td>0.035</td>
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</table>
Table 4: Credit Rating Downgrades and Market-Based Measures of Price Informativeness

Table 4 presents the results of the regression estimating the change in market-based measures of stock price informativeness following a credit rating downgrade. We estimate the following regression in each matched sample focusing on two quarters before the event to 4 quarters after:

\[ y_{it} = \beta_0 + \sum_{t=0}^{4} \lambda_t \cdot Treated_i + \sum_{t=0}^{4} \theta_t \cdot \delta_t + \mu_1 \cdot \delta_{-2} + \mu_2 \cdot \delta_{-2} \cdot Treated_i + \phi \cdot Treated_i + \Delta'X_{it-1} + \alpha_i + \gamma_t + q_t + \epsilon_{it} \]

where \( y_{it} \) is one of Non-synchronicity or PIN. \( Treated_i \) is an indicator variable that takes the value of one if the issuer experienced a rating downgrade, and it is zero otherwise. \( \delta_t \) is a indicator variables that takes the value of one if the time period is \( t \) quarters before/after the downgrade, and it is zero otherwise. \( X_{it-1} \) is a set of controls. \( \alpha_i, \gamma_t \) and \( q_t \) are firm, year and quarter fixed effects, respectively. The coefficients associated with the fixed effects or with the time indicators, \( \delta \) are not reported for brevity. Standard errors are clustered at the firm level. All variables are winsorized at the 1th and the 99th percentile. Variables definition appear in the Appendix.

<table>
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<tr>
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<th>CS2</th>
<th>Multiple Notch Downgrades</th>
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<th>CS2</th>
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<td>PIN</td>
<td>Non Synchronicity</td>
<td>PIN</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
</tr>
<tr>
<td>Treated*(\delta_{-2})</td>
<td>-.005</td>
<td>-.004</td>
<td>.0008</td>
<td>.0008</td>
<td>-.0009</td>
</tr>
<tr>
<td></td>
<td>(.005)</td>
<td>(.006)</td>
<td>(.003)</td>
<td>(.003)</td>
<td>(.009)</td>
</tr>
<tr>
<td>Treated*(\delta_0)</td>
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<td>.021</td>
<td>.005</td>
<td>.005</td>
<td>.035</td>
</tr>
<tr>
<td></td>
<td>(.006)**</td>
<td>(.006)**</td>
<td>(.003)*</td>
<td>(.003)*</td>
<td>(.009)**</td>
</tr>
<tr>
<td>Treated*(\delta_1)</td>
<td>.033</td>
<td>.029</td>
<td>.002</td>
<td>.003</td>
<td>.060</td>
</tr>
<tr>
<td></td>
<td>(.006)**</td>
<td>(.006)**</td>
<td>(.003)</td>
<td>(.003)</td>
<td>(.012)**</td>
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<tr>
<td>Treated*(\delta_2)</td>
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<td>.026</td>
<td>.006</td>
<td>.007</td>
<td>.054</td>
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<tr>
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<td>(.007)**</td>
<td>(.003)**</td>
<td>(.003)**</td>
<td>(.015)**</td>
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<tr>
<td>Treated*(\delta_3)</td>
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<td>.002</td>
<td>.047</td>
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<tr>
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<td>(.007)**</td>
<td>(.007)**</td>
<td>(.003)</td>
<td>(.003)</td>
<td>(.014)**</td>
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<td>Treated*(\delta_4)</td>
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<td>.004</td>
<td>.003</td>
<td>.058</td>
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<td></td>
<td>(.007)**</td>
<td>(.007)**</td>
<td>(.003)</td>
<td>(.003)</td>
<td>(.015)**</td>
</tr>
<tr>
<td>Size_{it-1}</td>
<td>-.050</td>
<td>-.016</td>
<td>-.016</td>
<td>-.031</td>
<td>-.015</td>
</tr>
<tr>
<td></td>
<td>(.011)**</td>
<td>(.003)***</td>
<td>(.028)</td>
<td>(.007)**</td>
<td></td>
</tr>
<tr>
<td>Turnover_{it-1}</td>
<td>-.0002</td>
<td>-.0005</td>
<td>-.0005</td>
<td>.0001</td>
<td>-.0006</td>
</tr>
<tr>
<td></td>
<td>(.0002)</td>
<td>(.00009)**</td>
<td>(.0004)</td>
<td>(.0002)**</td>
<td></td>
</tr>
<tr>
<td>Div. Yield_{it-1}</td>
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<td>-.031</td>
<td>-.031</td>
<td>-.017</td>
<td>-.018</td>
</tr>
<tr>
<td></td>
<td>(.079)</td>
<td>(.028)</td>
<td>(.141)</td>
<td>(.063)</td>
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<tr>
<td>Mkt. to Book_{it-1}</td>
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<td>-.00007</td>
<td>-.00007</td>
<td>-.00009</td>
<td>-.0002</td>
</tr>
<tr>
<td></td>
<td>(.0004)</td>
<td>(.0002)</td>
<td>(.0007)</td>
<td>(.0004)</td>
<td></td>
</tr>
<tr>
<td>Return_{it-1}</td>
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<td>-.002</td>
<td>-.002</td>
<td>.013</td>
<td>-.003</td>
</tr>
<tr>
<td></td>
<td>(.003)**</td>
<td>(.002)</td>
<td>(.005)**</td>
<td>(.003)</td>
<td></td>
</tr>
<tr>
<td>Price_{it-1}</td>
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<td>-.0009</td>
<td>.00009</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.0002)**</td>
<td>(.00006)</td>
<td>(.0004)**</td>
<td>(.0001)</td>
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<td>.126</td>
<td>.265</td>
<td>.805</td>
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<td></td>
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<td>(.089)**</td>
<td>(.005)**</td>
<td>(.028)**</td>
<td>(.017)**</td>
</tr>
<tr>
<td>Obs.</td>
<td>28208</td>
<td>28208</td>
<td>15684</td>
<td>15684</td>
<td>9235</td>
</tr>
<tr>
<td>(R^2)</td>
<td>.82</td>
<td>.821</td>
<td>.598</td>
<td>.6</td>
<td>.815</td>
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</table>
Table 5: Credit Rating Downgrades and Ownership-Based Measures of Price Informative-

Table 5 presents the results of the regression estimating the change in ownership-based measures of stock price informativeness following a credit rating downgrade. We estimate the following regression in each matched sample focusing on two quarters before the event to 4 four quarters after:

\[ y_{it} = \beta_0 + \sum_{t=0}^{4} \lambda_t \delta_t \cdot Treated_i + \sum_{t=0}^{4} \theta_t \delta_t + \mu_1 \delta_{-2} + \mu_2 \delta_{-2} \cdot Treated_i + \phi \cdot Treated_i + \Delta' X_{it-1} + \alpha_i + \gamma_t + \eta_t + \epsilon_{it} \]

where \( y_{it} \) is one of Active Mutual Fund Ownership or Long Term Institution Ownership. \( Treated_i \) is an indicator variable that takes the value of one if the issuer experienced a rating downgrade, and it is zero otherwise. \( \delta_t \) is a indicator variables that takes the value of one if the time period is \( t \) quarters before/after the downgrade, and it is zero otherwise. \( X_{it-1} \) is a set of controls. \( \alpha_i, \gamma_t \) and \( \eta_t \) are firm, year and quarter fixed effects, respectively. The coefficients associated with the fixed effects or with the time indicators, \( \delta \) are not reported for brevity. Standard errors are clustered at the firm level. All variables are winsorized at the 1th and the 99th percentile. Variables definition appear in the Appendix.

<table>
<thead>
<tr>
<th>Matched Sample</th>
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<th>Multiple Notch Downgrades</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dep. Variable</td>
<td>Active Mutual</td>
<td>Long Term</td>
</tr>
<tr>
<td></td>
<td>(1) (2) (3) (4)</td>
<td>(5) (6) (7) (8)</td>
</tr>
<tr>
<td>Treated*δ_{-2}</td>
<td>.003 .004 .002 .002</td>
<td>.007 .007 .002 .002</td>
</tr>
<tr>
<td></td>
<td>(.012) (.012) (.003) (.003)</td>
<td>(.023) (.023) (.006) (.006)</td>
</tr>
<tr>
<td>Treated*δ_{0}</td>
<td>.015 .012 .0008 -.0001</td>
<td>.045 .046 .002 .001</td>
</tr>
<tr>
<td></td>
<td>(.012) (.012) (.003) (.003)</td>
<td>(.024) (.024) (.007) (.007)</td>
</tr>
<tr>
<td>Treated*δ_{1}</td>
<td>.007 .001 .004 .002</td>
<td>.019 .016 .019 .018</td>
</tr>
<tr>
<td></td>
<td>(.013) (.013) (.004) (.004)</td>
<td>(.030) (.030) (.010) (.010)** (.009)**</td>
</tr>
<tr>
<td>Treated*δ_{2}</td>
<td>.012 .008 .004 .002</td>
<td>-.005 -.004 .019 .019</td>
</tr>
<tr>
<td></td>
<td>(.014) (.014) (.005) (.005)</td>
<td>(.030) (.030) (.011) (.011)</td>
</tr>
<tr>
<td>Treated*δ_{3}</td>
<td>.008 .005 .001 -.001</td>
<td>.019 .022 .012 .012</td>
</tr>
<tr>
<td></td>
<td>(.013) (.013) (.005) (.005)</td>
<td>(.029) (.029) (.012) (.012)</td>
</tr>
<tr>
<td>Treated*δ_{4}</td>
<td>.009 .006 -.006 -.008</td>
<td>-.002 -.005 -.003 -.003</td>
</tr>
<tr>
<td></td>
<td>(.014) (.014) (.005) (.005)</td>
<td>(.028) (.029) (.012) (.012)</td>
</tr>
<tr>
<td>Size_{t-1}</td>
<td>-.0003 -.012 .047 -.020</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.016) (.008) (.035) (.014)</td>
<td></td>
</tr>
<tr>
<td>Turnover_{t-1}</td>
<td>.002 -.0003 .002 -.0001</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.0005)*** (.0002) (.001)* (.0004)</td>
<td></td>
</tr>
<tr>
<td>Div. Yield_{t-1}</td>
<td>.592 .039 1.172 .095</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.184)*** (.060) (.412)*** (.136)</td>
<td></td>
</tr>
<tr>
<td>Mkt. to Book_{t-1}</td>
<td>-.0004 -.0004 -.003 -.00002</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.0008) (.0003) (.003) (.0007)</td>
<td></td>
</tr>
<tr>
<td>Return_{t-1}</td>
<td>-.006 .006 -.015 -.007</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.099) (.003)** (.019) (.006)</td>
<td></td>
</tr>
<tr>
<td>Price_{t-1}</td>
<td>-.0008 -.0005 .0002 -.0002</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.0003)** (.0001)*** (.0006) (.0002)</td>
<td></td>
</tr>
<tr>
<td>Const.</td>
<td>.215 .214 .250 .368 .259 -.153 .241 .402</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.025)*** (.141) (.007)*** (.062)*** (.064)*** (.291) (.013)*** (.113)***</td>
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</tr>
<tr>
<td>Obs.</td>
<td>14461 14461 21524 21524 3758 3758 6614 6614</td>
<td></td>
</tr>
<tr>
<td>R^2</td>
<td>.451 .454 .749 .75 .471 .476 .751 .752</td>
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</tr>
</tbody>
</table>
Table 6 presents the results of the regression estimating the change in market-based measures of stock price informativeness following a credit rating upgrade. We estimate the following regression in each matched sample focusing on two quarters before the event to 4 quarters after:

$$y_{it} = \beta_0 + \sum_{t=0}^{4} \lambda_t \cdot \delta_t \cdot Treated_i + \sum_{t=0}^{4} \theta_t \cdot \delta_t + \mu_1 \cdot \delta_{-2} + \mu_2 \cdot \delta_{-2} \cdot Treated_i + \phi \cdot Treated_i + \Delta'X_{it-1} + \alpha_i + \gamma_t + q_t + \epsilon_{it}$$

where $y_{it}$ is one of Non-synchronicity or PIN. $Treated_i$ is an indicator variable that takes the value of one if the issuer experienced a rating downgrade, and it is zero otherwise. $\delta_t$ is an indicator variable that takes the value of one if the time period is $t$ quarters before/after the downgrade, and it is zero otherwise. $X_{it-1}$ is a set of controls. $\alpha_i$, $\gamma_t$ and $q_t$ are firm, year and quarter fixed effects, respectively. The coefficients associated with the fixed effects or with the time indicators, $\delta$ are not reported for brevity. Standard errors are clustered at the firm level. All variables are winsorized at the 1th and the 99th percentile. Variables definition appear in the Appendix.

<table>
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<th>Matched Sample</th>
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<th>Multiple Notch Upgrades</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dep. Variable</td>
<td>Non Synchronicity</td>
<td>PIN</td>
</tr>
<tr>
<td>Treated$\delta_{-2}$</td>
<td>-0.008</td>
<td>-0.008</td>
</tr>
<tr>
<td>(0.007)</td>
<td>(0.007)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Treated$\delta_0$</td>
<td>0.002</td>
<td>0.003</td>
</tr>
<tr>
<td>(0.007)</td>
<td>(0.007)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Treated$\delta_1$</td>
<td>-0.010</td>
<td>-0.007</td>
</tr>
<tr>
<td>(0.008)</td>
<td>(0.008)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Treated$\delta_2$</td>
<td>-0.017</td>
<td>-0.013</td>
</tr>
<tr>
<td>(0.009)</td>
<td>(0.009)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Treated$\delta_3$</td>
<td>-0.016</td>
<td>-0.011</td>
</tr>
<tr>
<td>(0.009)</td>
<td>(0.009)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Treated$\delta_4$</td>
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<td>-0.023</td>
</tr>
<tr>
<td>(0.009)</td>
<td>(0.009)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Size$_{-1}$</td>
<td>-0.027</td>
<td>-0.008</td>
</tr>
<tr>
<td>(0.010)</td>
<td>(0.005)</td>
<td>(0.024)</td>
</tr>
<tr>
<td>Turnover$_{-1}$</td>
<td>-0.002</td>
<td>-0.008</td>
</tr>
<tr>
<td>(0.004)</td>
<td>(0.001)</td>
<td>(0.009)</td>
</tr>
<tr>
<td>Div. Yield$_{-1}$</td>
<td>0.178</td>
<td>0.05</td>
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<tr>
<td>(0.164)</td>
<td>(0.73)</td>
<td>(0.448)</td>
</tr>
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<td>Mkt. to Book$_{-1}$</td>
<td>-0.003</td>
<td>-0.001</td>
</tr>
<tr>
<td>(0.003)</td>
<td>(0.001)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>Return$_{-1}$</td>
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<td>0.03</td>
</tr>
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<td>(0.006)</td>
<td>(0.02)</td>
<td>(0.014)</td>
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<tr>
<td>Price$_{-1}$</td>
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<td>0.0007</td>
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<tr>
<td>(0.002)</td>
<td>(0.0005)</td>
<td>(0.005)</td>
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<td>19412</td>
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<tr>
<td>$R^2$</td>
<td>0.787</td>
<td>0.79</td>
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Table 7: Credit Rating Upgrades and Ownership-Based Measures of Price Informativeness

Table 7 presents the results of the regression estimating the change in ownership-based measures of stock price informativeness following a credit rating upgrade. We estimate the following regression in each matched sample focusing on two quarters before the event to 4 quarters after:

\[ \begin{align*}
    y_{it} &= \beta_0 + \sum_{t=0}^{4} \lambda_t \cdot \delta_t \cdot \text{Treated}_i + \sum_{t=0}^{4} \theta_t \cdot \delta_t + \mu_1 \cdot \delta_{-2} + \mu_2 \cdot \delta_{-2} \cdot \text{Treated}_i + \phi \cdot \text{Treated}_i \\
    &+ \Delta' X_{it-1} + \alpha_i + \gamma_t + q_t + \epsilon_{it}
\end{align*} \]

where \( y_{it} \) is one of \textit{Active Mutual Fund Ownership} or \textit{Long Term Institution Ownership}. \text{Treated}_i is an indicator variable that takes the value of one if the issuer experienced a rating downgrade, and it is zero otherwise. \( \delta_t \) is a indicator variables that takes the value of one if the time period is \( t \) quarters before/after the downgrade, and it is zero otherwise. \( X_{it-1} \) is a set of controls. \( \alpha_i \), \( \gamma_t \) and \( q_t \) are firm, year and quarter fixed effects, respectively. The coefficients associated with the fixed effects or with the time indicators, \( \delta \) are not reported for brevity. Standard errors are clustered at the firm level. All variables are winsorized at the 1th and the 99th percentile. Variables definition appear in the Appendix.

<table>
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<tr>
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<th></th>
</tr>
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<tbody>
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<td></td>
<td>CS1</td>
<td>CS2</td>
<td>CS1</td>
<td>CS2</td>
</tr>
<tr>
<td>Treated*( \delta_{-2} )</td>
<td>.004</td>
<td>.006</td>
<td>.0007</td>
<td>.003</td>
</tr>
<tr>
<td></td>
<td>(.012)</td>
<td>(.012)</td>
<td>(.003)</td>
<td>(.003)</td>
</tr>
<tr>
<td>Treated*( \delta_0 )</td>
<td>-0.11</td>
<td>-0.10</td>
<td>-0.03</td>
<td>-0.03</td>
</tr>
<tr>
<td></td>
<td>(.012)</td>
<td>(.012)</td>
<td>(.003)</td>
<td>(.003)</td>
</tr>
<tr>
<td>Treated*( \delta_1 )</td>
<td>-0.04</td>
<td>-0.04</td>
<td>-0.02</td>
<td>-0.02</td>
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<tr>
<td></td>
<td>(.012)</td>
<td>(.012)</td>
<td>(.004)</td>
<td>(.004)</td>
</tr>
<tr>
<td>Treated*( \delta_2 )</td>
<td>-0.16</td>
<td>-0.14</td>
<td>-0.04</td>
<td>-0.04</td>
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<tr>
<td></td>
<td>(.014)</td>
<td>(.014)</td>
<td>(.005)</td>
<td>(.005)</td>
</tr>
<tr>
<td>Treated*( \delta_3 )</td>
<td>-0.17</td>
<td>-0.15</td>
<td>-0.03</td>
<td>-0.03</td>
</tr>
<tr>
<td></td>
<td>(.014)</td>
<td>(.014)</td>
<td>(.005)</td>
<td>(.005)</td>
</tr>
<tr>
<td>Treated*( \delta_4 )</td>
<td>-0.18</td>
<td>-0.15</td>
<td>-0.07</td>
<td>-0.07</td>
</tr>
<tr>
<td></td>
<td>(.014)</td>
<td>(.014)</td>
<td>(.005)</td>
<td>(.005)</td>
</tr>
<tr>
<td>Size_{t-1}</td>
<td>-0.26</td>
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<td>-0.09</td>
<td>-0.09</td>
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<tr>
<td></td>
<td>(.17)</td>
<td>(.007)</td>
<td>(.037)</td>
<td>(.037)</td>
</tr>
<tr>
<td>Turnover_{t-1}</td>
<td>.003</td>
<td>-0.004</td>
<td>.003</td>
<td>-0.009</td>
</tr>
<tr>
<td></td>
<td>(.285)</td>
<td>(.129)</td>
<td>(1.063)</td>
<td>(.423)</td>
</tr>
<tr>
<td>Div. Yield_{t-1}</td>
<td>-0.434</td>
<td>.078</td>
<td>1.195</td>
<td>.365</td>
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<td></td>
<td>(.285)</td>
<td>(.129)</td>
<td>(1.063)</td>
<td>(.423)</td>
</tr>
<tr>
<td>Mkt. to Book_{t-1}</td>
<td>-0.0002</td>
<td>.0001</td>
<td>.0007</td>
<td>-0.0001</td>
</tr>
<tr>
<td></td>
<td>(.0004)</td>
<td>(.0001)</td>
<td>(.0006)</td>
<td>(.0003)</td>
</tr>
<tr>
<td>Return_{t-1}</td>
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<td>.003</td>
<td>-0.004</td>
<td>-0.003</td>
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<tr>
<td></td>
<td>(.011)</td>
<td>(.004)</td>
<td>(.037)</td>
<td>(.013)</td>
</tr>
<tr>
<td>Price_{t-1}</td>
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<td>-0.0003</td>
<td>-0.0005</td>
<td>-0.0003</td>
</tr>
<tr>
<td></td>
<td>(.0002)</td>
<td>(.00007)**</td>
<td>(.0008)</td>
<td>(.0002)</td>
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<td>Const.</td>
<td>.316</td>
<td>.525</td>
<td>.222</td>
<td>.161</td>
</tr>
<tr>
<td></td>
<td>(.024)**</td>
<td>(.143)**</td>
<td>(.006)**</td>
<td>(.005)**</td>
</tr>
<tr>
<td>Obs.</td>
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<td>13080</td>
<td>16793</td>
<td>16793</td>
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<tr>
<td>( R^2 )</td>
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<td>.451</td>
<td>.756</td>
<td>.757</td>
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Appendix B: Proofs

Lemma 1. In the market clearing condition (6), the only demand that endogenously depend on the learning from price is the one of rational uninformed investors. Taking the conjectured linear price function as given, the signal extracted by any rational investor that learns from price is

$$s_1 = \frac{P}{\beta} = \tilde{v} + \frac{\gamma}{\beta} \tilde{e} + \frac{\eta}{\beta} \tilde{z}$$

With that signal, rational uninformed investors update (in a bayesian way) their valuation for the asset to

$$E[v|P] = \frac{\beta \sigma_v^2}{(\beta^2 \sigma_v^2 + \gamma^2 \sigma^2_e + \eta^2 \sigma^2_z)}$$

Replacing this expected value into the market clearing condition, we get:

$$P \left(1 - \frac{\beta \sigma^2 (1 - \alpha)(1 - \lambda)}{(\beta^2 \sigma_v^2 + \gamma^2 \sigma^2_e + \eta^2 \sigma^2_z)}\right) = \alpha \tilde{v} + (1 - \alpha) \lambda \tilde{z} + c \tilde{e}$$

Matching terms from this last equation to the three coefficients in the conjectured linear pricing rule of equilibrium, we obtain that

$$\beta = \frac{\alpha (\beta^2 \sigma_v^2 + \gamma^2 \sigma^2_e + \eta^2 \sigma^2_z)}{(\beta^2 \sigma_v^2 + \gamma^2 \sigma^2_e + \eta^2 \sigma^2_z) - \beta \sigma_v^2 (1 - \alpha)(1 - \lambda)}$$

$$\gamma = \frac{c (\beta^2 \sigma_v^2 + \gamma^2 \sigma^2_e + \eta^2 \sigma^2_z)}{(\beta^2 \sigma_v^2 + \gamma^2 \sigma^2_e + \eta^2 \sigma^2_z) - \beta \sigma_v^2 (1 - \alpha)(1 - \lambda)}$$

$$\gamma = \frac{(1 - \alpha) \lambda (\beta^2 \sigma_v^2 + \gamma^2 \sigma^2_e + \eta^2 \sigma^2_z)}{(\beta^2 \sigma_v^2 + \gamma^2 \sigma^2_e + \eta^2 \sigma^2_z) - \beta \sigma_v^2 (1 - \alpha)(1 - \lambda)}$$

From there, it is easy to see that for a given \(\alpha\), the coefficients in the pricing rule must satisfy: \(\frac{\gamma}{\beta} = \frac{c}{\alpha}\), and \(\frac{\eta}{\beta} = \frac{(1 - \alpha)}{\alpha} \lambda\). Replacing these relationships into the equation obtained for \(\beta\) in equilibrium, we finally obtain that:

$$\beta = \frac{\alpha^2 (\sigma_v)^2 (1 - \lambda (1 - \alpha)) + \alpha (c^2 \sigma^2_e + (1 - \alpha)^2 \lambda^2 \sigma^2_z)}{\alpha^2 (\sigma_v)^2 + c^2 \sigma^2_e + (1 - \alpha)^2 \lambda^2 \sigma^2_z}$$

Finally, for a given \(\alpha\), the solution to this system of equations is unique, whenever it exists, which is sufficient to ensure uniqueness of the trading equilibrium when price is a linear rule. ■
Lemma 2. At \( t = 0 \), anticipating a change in rating at \( t = 1 \), we first calculate the expected utility of an investor that considers both alternatives - becoming informed or remaining uniformed. The expected utility of an informed investor at \( t = 0 \) would be

\[
E_0[W_i] = E_0[W_{i0} + X_0^I(v - P) + \frac{(v - P)^2}{c} - \frac{c}{2} \frac{(v - P)^2}{c^2} - k]
\]

It is easy to see that \( E_0[(v - P)] = 0 \). Also, replacing the conjectured linear pricing rule, we can see that \( E_0[(v - P)^2] = \sigma_v^2(1 - \beta)^2 + \gamma^2 \sigma_x^2 + \eta^2 \sigma_z^2 \). With this, the expected utility for an informed investor at \( t = 0 \) becomes

\[
B_i0 - k + \frac{1}{2c} \sigma_v^2(1 - \beta)^2 + \gamma^2 \sigma_x^2 + \eta^2 \sigma_z^2).
\]

Similarly, for a rational uninformed investor, expected utility at \( t = 0 \) is

\[
E_0[W_{RU_i}] = E_0[W_{i0} + X_{RU_0}(\mu_P - P) + \frac{(\mu_P - P)^2}{c} - \frac{c}{2} \frac{(\mu_P - P)^2}{c^2}],
\]

where \( \mu_P \equiv E[v|P] \). The reader can check that \( E[(\mu_P - P)] = 0 \). Also,

\[
(\mu_P - P)^2 = \frac{\sigma_v^4(1 - \beta)^2 + \gamma^2 \sigma_x^2 + \eta^2 \sigma_z^2 - 2\sigma_v^2(1 - \beta)\beta(\gamma^2 \sigma_x^2 + \eta^2 \sigma_z^2)}{(\beta^2 \sigma_v^2 + \gamma^2 \sigma_x^2 + \eta^2 \sigma_z^2)^2}\]

Hence, taking expectation at \( t = 0 \) we have that

\[
E_0[(\mu_P - P)^2] = \frac{\sigma_v^4 \beta^2}{(\beta^2 \sigma_v^2 + \gamma^2 \sigma_x^2 + \eta^2 \sigma_z^2)} + (\beta^2 \sigma_v^2 + \gamma^2 \sigma_x^2 + \eta^2 \sigma_z^2) - 2\sigma_v^2 \beta.
\]

Replacing back, we finally get that the expected utility of an uninformed investor that ends up being rational at \( t = 1 \) is

\[
E_0[W_{RU_1}] = B_{i0} + \frac{1}{2c} \left[ \frac{\sigma_v^4 \beta^2}{(\beta^2 \sigma_v^2 + \gamma^2 \sigma_x^2 + \eta^2 \sigma_z^2)} + (\beta^2 \sigma_v^2 + \gamma^2 \sigma_x^2 + \eta^2 \sigma_z^2) - 2\sigma_v^2 \beta \right]
\]

The expected utility for an incorrect uninformed investor is calculated in a similar way. Now, if a fraction \( \alpha^* \) of informed investors in the market exists (not trivially equal to one or zero), it must be the case that a marginal investor, indifferent between becoming informed at \( t = 0 \) or not, exists. The indifference for such marginal investor requires the following
condition to be satisfied:

\[ B_{i0} - k + \frac{1}{2c}(\sigma^2 v(1 - \beta)^2 + \gamma^2 \sigma^2_z + \eta^2 \sigma^2_z) = (1 - \lambda) \left[ B_{i0} + \frac{1}{2c} \left( \frac{\sigma^4 \beta^2}{(\beta^2 \sigma^2_z + \gamma^2 \sigma^2_z + \eta^2 \sigma^2_z)} + (\beta^2 \sigma^2_v + \gamma^2 \sigma^2_z + \eta^2 \sigma^2_z) - 2\sigma^2 \beta \right) \right] + \lambda \left[ B_{i0} + \frac{1}{2c}(\beta^2 \sigma^2_v + \gamma^2 \sigma^2_z + \eta^2 \sigma^2_z - 2\beta \sigma^2 - \sigma^2_z) \right] \]

After some algebra, this condition becomes:

\[-2kc + \sigma^2 v + \lambda \sigma^2_z = \frac{(1 - \lambda)\sigma^4 \beta^2}{(\beta^2 \sigma^2_v + \gamma^2 \sigma^2_z + \eta^2 \sigma^2_z)}\]

Using the already established relationships between \(\beta\) and the other two coefficients in the pricing rule, we can rewrite this condition in terms of \(\beta\) only, and see that some terms cancel out, ending with the following condition:

\[ (-2kc + \sigma^2_v + \lambda \sigma^2_z)(\sigma^2_v c^2 \sigma^2_v \alpha^2 \sigma^2 \sigma^2 + \frac{(1 - \alpha)2\lambda^2}{\alpha^2} \sigma^2_z) = (1 - \lambda)\sigma^4 \]

Rearranging terms, we obtain a quadratic equation for \(\alpha^*\), which solution corresponds to the expression stated in the lemma. It can be checked that the other potential solution to the above equation always yields \(\alpha \geq 1\), which is not the scenario of interest as part of the equilibrium that we are analyzing. ■

Proposition 1. Let’s first define \(N(\omega) \equiv \frac{(1 - \lambda)\sigma^4 \beta^2}{(\sigma^2_v + \lambda \sigma^2_z - \omega)}\). Next, it can be checked that \(\omega \equiv \omega(k)\) corresponds to the value of \(\omega\) (or \(k\), respectively) that makes zero the discriminant of the quadratic equation determining the possible solutions to it. Hence, for \(\omega\), the quadratic equation has a unique solution \(\alpha^*(\omega)\). It can be checked that \(\omega = \lambda \sigma^2_z + \theta \sigma^2_v\), consistent with how it was previously defined. Replacing this value of \(\omega\) in equation (8), we can see that \(\alpha^*(\omega) > 1\), which makes it unfeasible as part of the equilibria that we are looking for. Given the monotonicity of the discriminant in the mentioned quadratic equation that determines \(\alpha^*\), we see that this discriminant is strictly positive for \(\omega > \omega\). On the other hand, it can be easily seen that \(N(\omega) > 0\) if \(\omega < \omega\). Going back to the discriminant of the quadratic equation for \(\alpha^*\), it can then be concluded that another necessary condition for a solution \(\alpha^*\) to exist is that \(\omega < \omega\) (otherwise the discriminant takes a negative value). Since \(\theta \leq 1\), we know that \(\omega \leq \omega\) (the equality is achieved only for the extreme case of \(\lambda = 1\)). Hence, at this point we are able to state that a necessary condition for an \(\alpha^* \in (0,1)\) of equilibrium to exist is that \(\omega \in (\omega, \omega)\). Furthermore, it can be easily checked that \(\alpha^* \rightarrow 0\)
in the limit as $\omega \to \overline{\omega}$, and that $\alpha^*$ monotonically decreases with $N$ (and hence it also decreases with $\omega$ and $k$). As noted the above condition on the value of $\omega$ is necessary, but it is fairly easy to see that it is not sufficient (by continuity and monotonicity, there are values of $\omega$ in that mentioned range for which $\alpha^* > 1$). Replacing $N(\omega)$ in equation (8), we can look for $N^*$ that makes $\alpha^* = 1$. This leads to a fixed point problem on $N^*$, characterized by the equation

$$N^* = \sigma_v^2 + \sqrt{(c^2 \sigma_v^2 + \lambda^2 \sigma_z^2)N - \lambda^2 \sigma_z^2 \sigma_v^2 - c^2 \sigma_z^2 (\sigma_v^2 + \lambda^2 \sigma_z^2)}.$$}

The concavity of the right hand side in the equation ensures that a solution to this fixed point exists (and it is unique) for $N \geq 0$. It also can be verified that for $N(\omega)$ the right hand side in the above equation is greater than the left hand side which, together with the fact that $\lim_{\omega \to \overline{\omega}} N(\omega) = \infty$, ensure that $\exists! N^* \in (N(\omega), N(\overline{\omega}))$ such that $\alpha^*(N^*) = 1$. Since $\alpha^*$ monotonically decreases in $N$, we have that $\alpha^* \in (0, 1)$ iff $\omega \in (\omega^*, \overline{\omega})$, where $\omega^*$ is the such that $N(\omega^*) = N^*$.

**Lemma 3.** Taking derivative of $\alpha^*$ with respect to $\sigma_v^2$. To simplify the notation in the next steps, let’s call

$$SQ = \sqrt{(c^2 \sigma_v^2 + \lambda^2 \sigma_z^2)N - \lambda^2 \sigma_z^2 \sigma_v^2 - c^2 \sigma_z^2 (\sigma_v^2 + \lambda^2 \sigma_z^2)},$$

$$V = \sigma_v^2$$

$$G = \frac{-1}{2SQ(V\lambda \sigma_z^2 - N)^2}$$

With that, we have

$$\frac{\partial \alpha^*}{\partial \sigma_v^2} = G \left( \frac{\partial N}{\partial V} - 1 \right) \left[ (c^2 \sigma_v^2 + \lambda^2 \sigma_z^2)(V + \lambda \sigma_z^2 - N) + (\lambda^2 \sigma_z^2 - SQ) \right]$$

Going back to the expression of $\alpha^*$, the last term in this equation can be rewritten as

$$\frac{\partial \alpha^*}{\partial \sigma_v^2} = G \left( \frac{\partial N}{\partial V} - 1 \right) \left[ (c^2 \sigma_v^2 + \lambda^2 \sigma_z^2)(V + \lambda \sigma_z^2 - N) + \alpha^*(V + \lambda \sigma_z^2 - N) \right].$$
which finally yields

$$\frac{\partial \alpha^*}{\partial \sigma_z^2} = G \left( \frac{\partial N}{\partial V} - 1 \right) (V + \lambda^2 \sigma_z^2 - N)(c^2 \sigma_z^2 + \lambda^2 \sigma_z^2 + \alpha^*).$$

It can be easily checked that for the conditions that ensure the existence of an overall equilibrium $G < 0$, and that the last term multiplying is necessarily strictly positive. Furthermore,

$$\frac{\partial N}{\partial V} = (1 - \lambda) \left[ 1 - \left( \frac{\lambda \sigma_z^2 - \omega}{V + \lambda \sigma_z^2 - \omega} \right) \right].$$

This derivative will be less than or equal to one in most of cases. A sufficient condition for that is that $\omega \leq \lambda \sigma_z^2$. Therefore, we now proceed to examine when does this condition hold, beginning with the case of $\omega^*$ (the lowest possible value of $\omega$). Replacing $\alpha^* = 1$ in the equation characterizing $\alpha^*$, we can solve for $\omega^*$, obtaining

$$\omega^* = V + \lambda \sigma_z^2 - \frac{(1 - \lambda)V^2}{c^2 \sigma_z^2 + V}.$$

It can then be seen that $\lambda \sigma_z^2 - \omega^* = -V(\lambda - c^2 \sigma_z^2)$. Evaluating this expression at $\lambda = 0$, we see that it takes a positive value. This ensures that $\frac{\partial N}{\partial V} < 1$. At the same time, we can check that $(V + \lambda \sigma_z^2 - N(\omega^*)) = \lambda \sigma_z^2 - c^2 \sigma_z^2$, which evaluated at $\lambda = 0$ takes a negative value, ensuring that $(V + \lambda \sigma_z^2 - N(\omega^*)) < 0$ at $\lambda = 0$. With all this, going back to the expression for $\frac{\partial \alpha^*}{\partial \sigma_z^2}$, we can observe that when $\omega = \omega^*$ and $\lambda = 0$, $\frac{\partial \alpha^*}{\partial \sigma_z^2} < 0$. Now, fixing $\lambda = 0$, we see that $\frac{\partial N}{\partial \lambda} > 0$, hence $(V + \lambda^2 \sigma_z^2 - N(\omega^*)) \bigg|_{\lambda=0} < 0 \ \forall \omega \in (\omega^*, \omega) = (0, \sigma_z^2)$. Then, we can fix $\omega = \omega^*$. It can be checked that a sufficient condition for $(\lambda \sigma_z^2 - \omega^*) > 0$ is that $\lambda < c^2 \sigma_z^2$. On the other hand, it can checked that $(V + \lambda^2 \sigma_z^2 - N^*) = \lambda^3 \sigma_z^2 - c^2 \sigma_z^2$. Thus, a sufficient condition for that term to be negative (as with $\lambda = 0$) is that $\lambda < (c^2 \sigma_z^2 / \sigma_z^2)^{1/3}$. With this, we can see that, for $\omega = \omega^*$ (and certainly a neighborhood around it), $\frac{\partial \alpha^*}{\partial \sigma_z^2} < 0 \ \forall \lambda \in (0, \min\{c^2 \sigma_z^2 / \sigma_z^2, (c^2 \sigma_z^2 / \sigma_z^2)^{1/3}\})$. For some other parameter combinations outside the specified regions, it is not possible to say much more about the sign of $\frac{\partial \alpha^*}{\partial \sigma_z^2}$ without making further assumptions. Nevertheless, it can be checked that as both $\lambda$ and $k$ grow larger, the sign of the mentioned derivative tends to switch sign.

**Proposition 2.** We mainly want to prove that $\frac{d\text{Var}}{dV} < 0$. To further ease notation, define $E = c^2 \sigma_z^2$, and $Z = \lambda^2 (\sigma_z)^2$. The we can rewrite the measure of price informativeness
\[
Var = V \left[ \frac{\varepsilon + (1 - \alpha)^2Z}{\alpha^2V + \varepsilon + (1 - \alpha)^2Z} \right]
\]

where \( \alpha \) is given by (8). In particular, \( \alpha \) can be replaced by defining \( N = \frac{\varepsilon + Z - 2\alpha v}{\alpha^2} + V + Z = \frac{\varepsilon + \alpha^2 V + (1 - \alpha)^2 Z}{\alpha^2} \). Hence,

\[
Var = \frac{V(N - V)}{N}
\]

\[
dVar = \frac{(N + V \frac{dN}{dV} - 2V)N - (VN - V^2) \frac{dN}{dV}}{N^2} = \frac{(N - 2V)N + V^2 \frac{dN}{dV}}{N^2}
\]

For the \( N \) and \( \frac{dN}{dV} \) in the numerator, we have that

\[
N = \frac{(1 - \lambda)\sigma^4_v}{\sigma^2 + \lambda \sigma^2_v - 2\kappa c} = \frac{(1 - \lambda)V^2}{V + \frac{Z}{\lambda} - 2\kappa c}
\]

\[
\frac{dN}{dV} = \frac{2(1 - \lambda)V(V + \frac{Z}{\lambda} - 2\kappa c) - (1 - \lambda)V^2}{(V + \frac{Z}{\lambda} - 2\kappa c)^2}
\]

\[
= \frac{(1 - \lambda)V(V + \frac{2Z}{\lambda} - 4\kappa c)}{(V + \frac{Z}{\lambda} - 2\kappa c)^2}
\]

Now the numerator in (16) is

\[
(N - 2V)N + V^2 \frac{dN}{dV} = \left[ \frac{(1 - \lambda)V^2}{V + \frac{Z}{\lambda} - 2\kappa c} - 2V \right] \cdot \frac{(1 - \lambda)V^2}{V + \frac{Z}{\lambda} - 2\kappa c} + V^2 \cdot \frac{(1 - \lambda)V(V + \frac{2Z}{\lambda} - 4\kappa c)}{(V + \frac{Z}{\lambda} - 2\kappa c)^2}
\]

\[
= \frac{(1 - \lambda)V^3}{(V + \frac{Z}{\lambda} - 2\kappa c)^2} \cdot \left[ (1 - \lambda)V - 2 (V + \frac{Z}{\lambda} - 2\kappa c) + (V + \frac{2Z}{\lambda} - 4\kappa c) \right]
\]

\[
= \frac{(1 - \lambda)V^3}{(V + \frac{Z}{\lambda} - 2\kappa c)^2} \cdot \left[ -\lambda V \right] < 0
\]

It is analogous to see that \( \frac{dVar}{d\lambda} < 0 \). Since with a downgrade in the model we experience both, a discrete increase in \( \lambda \) and in \( \sigma^2_v \), this proves that with a downgrade price informativeness should go up. Conversely, for an upgrade, \( \sigma^2_v \) and \( \lambda \) decrease, so we experience the opposite effects. As \( \lambda \) is lower in this case though, and incentives to become informed tend to be relatively lower, there is a counterbalancing effect of a decreasing fraction of informed investors. In such a case, the population of uninformed investors increases, thus increasing at the same time the expected size of noise trading coming from investors with limited attention. This makes the prediction that results from the model relatively weaker in the
case of an upgrade. Particularly, it can be checked that for levels of $\lambda \to 0$, the overall effect in an upgrade is a decrease in price informativeness (this is, making both, $\lambda(\sigma^2_v,0)$ and $\lambda(\sigma^2_v,0')$ be approximately zero). For larger values of $\lambda$ it can be checked that the overall effect can be lower depending on the combination of parameters under consideration.

Appendix C: Supporting the partial information acquisition of a CRA

Here, we provide a more detailed explanation of the model that we consider when adopting the reduced-form role of the representative CRA. Importantly, the argument relies on the lack of competition faced by CRAs in practice, and the limited resources that a CRA may have to acquire information about each individual firm for which a rating is provided. Thus, we assume that the representative CRA in our model behaves as a monopoly that jointly chooses the precision of the information collected about each firm, and the number of firms to rate at a given point in time.

Consider a risk neutral CRA, that maximizes profits. Consider that there are infinitely many firms to be rated, and that the CRA has a limited budget $W_0$. This budget must be invested in acquiring information (hiring analysts, investing in information technologies that allow for reliable estimations of the risks involved in each firm and industry, etc.). We assume that the CRA has two alternatives in terms of the precision of the information to be acquired about each rated firm:

- Option 1 is to be extremely precise in the ratings. This, in the context of the model in this paper, we assume involves spending $k$ to learn the actual realization $v$ of firm value.\(^{19}\) This would involve a rating that could potentially be fully accurate about the risk of default of the firm. In other words, if we denote $D$ the total value of debt obligations for that firm at that point in time, then the information that the CRA could potentially offer through the rating would be $P(v < D) \in \{0, 1\}$. Given the full precision of the rating in this case, we consistently assume that the fee that could be obtained by the CRA from one of these rating would be $f_f$ (higher than any other

\(^{19}\)In the main model, $\tilde{v}$ corresponds to firm value per share, but here, for simplicity, we abuse notation and denote $\tilde{v}$ the actual total firm value.
quality of rating that could be provided).

- Option 2 is to be just partially informative. Again in the context of the model in this paper, we assume that this case involves learning about the state of nature $\Omega$, which allows for a partially informative rating. This would actually imply that each rating would transmit a $P(\hat{\nu} < D) \in (0, 1)$. We can assume that the cost of obtaining this partial information is $m << k$, and that the fee that could be charged for this type of rating is $f_p << f_f$.

We assume that all costs and fees are the same for each firm. Therefore, in the process of maximizing profits, the CRA would never choose a mixed strategy where would get full information for some rated firms, and partial information for some others. Then, the comparison is between the profits that can be obtained by applying the same type of rating to all firms, comparing full versus partial information. It can be noted that, for a limited budget $W_0$, if the CRA gets full information about all firms being rated, it would be able to rate $n_f = W_0/k$ (which, for simplicity we assume is an integer value), whereas if all rating deliver partial information on the probability of default of firms, the total number of firms rated could be $n_p = W_0/m$. Clearly, by the assumption on the cost of acquiring information, $n_p >> n_f$. Thus, when comparing profits, we would have:

$$\pi_f = n_f(f_f - k) = W_0(f_f/k - 1) < \pi_p = n_p(f_p - m) = W_0(f_p/m - 1)$$

if and only if $f_f/f_p < k/m$. Though we are assuming $f_f/f_p > 1$, it is intuitive to think that in practice these fees are limited by either a potential regulator not allowing CRAs to charge extremely high fees from the firms that are being rated, or from the actual threat of creating room for a potential competitor to come into place. None of these factors are modeled here, but are supporting arguments in favor of having $f_f/f_p$ bounded from above. On the other hand, we can think of $m$ being very low relative to $k$, thanks to the expertise of analysts in learning about some partial information that is more standard across firms (state of natures affecting whole industries, for example), while collecting really private information (worth being traded) falls into a more specialized category, outside the objectives of a CRA, and hence much more costly to implement for each individual firm. Under the described scenario, we should expect $f_f/f_p < k/m$ to hold, and so a CRA choosing to collect partial
information about states of nature $\Omega$ to be used in the ratings of firms in equilibrium.

Lastly, it is worth mentioning that, an even more complete model of CRA, not only would consider some level of oligopolistic competition (which may be even more aligned to what the scenario just described, in terms of the relative fees that can be charged for each quality of rating), but, more importantly, would consider a dynamic setting, where reputation concerns play a role in determining the fees that can be charged by CRAs more directly. Under that case, being right on average (about which firms are more likely to default) would intuitively build reputation. It is intuitive to think that, with partial information ratings, but a very high number of ratings being provided, by the law of large numbers, rating should tend to be very accurate on average. Thus, CRAs would be able to generate a fairly similar effect in terms of reputation from delivering partial information ratings to a large number of firms or a fully revealing rating to a lower number of firms. This argument seems to favor the idea of the fees in both cases being not that different, and so, as long as the cost of acquiring partial information is low enough compared to acquiring the same level of information of a privately informed investor, partially informative ratings would still be supported in equilibrium.
References


